

ELECTRONIC TECHNOLOGY SERIES

No. 166-23

# IMPEDANCE MATCHING

a **RIDER** publication



# **IMPEDANCE MATCHING**

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# IMPEDANCE MATCHING

*edited by*

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**LIBRARY OF CONGRESS CATALOG CARD NUMBER 57-14996**

Printed in the United States of America

**FIFTH PRINTING 1963**

## PREFACE

Few topics must be given as close attention in the design and construction of electronic equipment as must impedance matching. In the fields of electricity and electronics we are constantly dealing with the transfer of energy from the output of one circuit to the input of another. A more common terminology refers to the energy transfer as taking place from a "source" or "generator" to a "load." The source may be any type of circuit or generator such as a microphone, phonograph cartridge, output transformer, or tuned circuit. The load might be a loudspeaker, grid circuit of a following stage, transmission line, or antenna. The function of impedance matching is to obtain maximum power transfer from the generator to the load.

This book is written as a guide to the engineer concerned with design considerations. The technician will find IMPEDANCE MATCHING an extremely valuable tool in his work in the repair and replacement of component parts. To the student is offered the theory involved in the transfer of power, which he must bear in mind in all future study of electrical and electronic circuitry. To the radio amateur, the hobbyist, and others interested in allied field this book represents the finest reference source on impedance matching available. To all concerned, IMPEDANCE MATCHING is a book to be read and used for future reference as a highly authoritative source of technical information on the subject.

Grateful acknowledgment is made to the staff of the New York Institute of Technology for its assistance in the preparation of the manuscript of the book.

*New York, N.Y.*  
*May 1958*

A.S.





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## Chapter 1

# IMPEDANCE MATCHING AND POWER TRANSFER

### 1. Definition of Impedance

One may safely make the statement that all electronic equipment either uses, converts, or wastes electrical power. Many times in the course of performing myriad tasks called upon by this equipment, power must be transferred from one device to another. The device from which we draw power is called the source and the device accepting this power is called the load. The efficiency of the transfer of power between this source and load is a function of their relative impedances. Impedance matching can therefore be defined as the adjustment of the source and the load impedances to produce the most efficient transfer of power from one to the other.

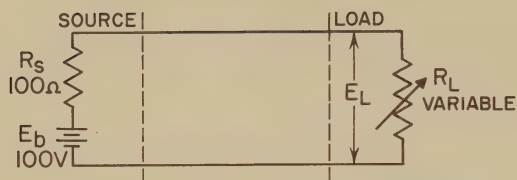
### 2. Maximum Power Transfer in D-C Circuits

The relation between impedance matching and power transfer for an electrical circuit can be illustrated by selecting a battery with a large internal resistance (calling this the source) and measuring the amount of power delivered to a load. Assume a 100-volt battery having an internal resistance of 100 ohms. The battery supplies power to a load of variable resistance. (See Fig. 1.)

Now, consider the power developed and the power delivered to the load resistor.

The power delivered to the load resistor  $R_L$  may be computed from the general formula for power:

$$\text{watts} = \frac{E^2}{R} \quad (1)$$



**Fig. 1.** Power transfer of a d-c circuit.

In this formula the usual symbols may be replaced with the designation given in Fig 1:

$$P_L \text{ (watts delivered to } R_L) = \frac{(E_L)^2}{R_L} \quad (2)$$

In the figure the voltage across the load resistor is proportional to the total resistance of the circuit. This may be written:

$$E_L = E_b \times \frac{R_L}{R_L + R_s} \quad (3)$$

Then the power delivered to  $R_L$  is:

$$P_L = \left( \frac{E_b \times R_L}{R_L + R_s} \right)^2 \frac{1}{R_L} = \frac{E_b^2 \times R_L}{(R_L + R_s)^2} \quad (4)$$

Substituting in formula 4 the values given in Fig. 1:

$$P_L = \frac{100^2 R_L}{(R_L + 100)^2} \quad (5)$$

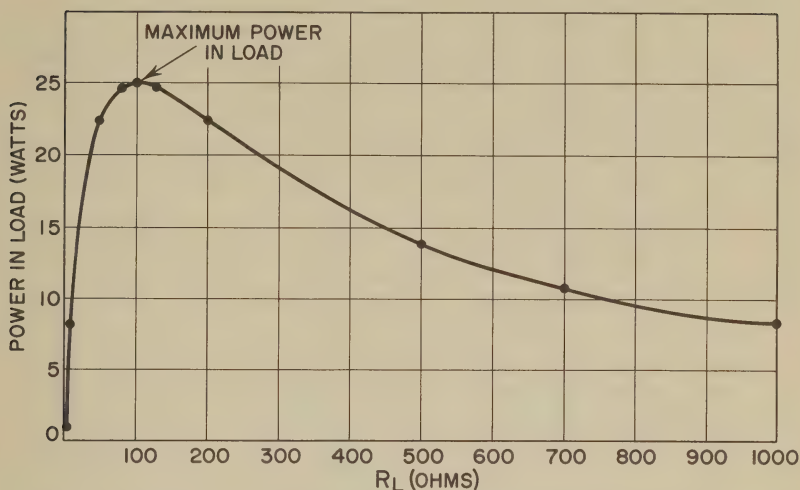
Solving formula 5 for various values of  $R_L$ , the results can be tabulated as follows:

**TABLE 1**

$R_L$ (ohms)	$P_T$ (total power-watts) (out of source)	$P_L$ (load power-watts) (into load)	$P_L/P_T \times 100$ percent of total power delivered
1	99.0	0.98	1.0
10	91.0	8.28	9.0
50	67.0	22.4	33.0
80	56.0	24.7	44.0
100	50.0	25.0	50.0
125	44.0	24.7	57.6
200	33.0	22.2	67.0
500	17.0	13.9	83.0
700	12.5	10.9	87.5
1000	9.0	8.3	92.1
10000	1.0	0.98	99.0

The values of power delivered against the values of  $R_L$  in ohms





**Fig. 2.** Graph shows the amount of power delivered, plotted against different values of load  $R_L$ .

are shown in Fig. 2. The greatest power is delivered when the load resistor has the same value as the source resistor.

The graph of power transfer is also drawn on a logarithmic or exponential scale which spreads out the lower values of the ohmic range of  $R_L$ .

Figure 3 shows, as did Fig. 2, that maximum power is delivered to the load resistor when the source matches the load, a condition occurring when both are 100 ohms. This is usually stated as a rule that should be memorized and recalled whenever this problem occurs. *The maximum amount of power is delivered to a load when the resistance of the load is equal to the resistance of the source.* This is more commonly referred to as *maximum power transfer*. Throughout this text we will often be concerned with circuit conditions requiring maximum power transfer.

The paradox of this situation is that when the greatest amount of power is being transferred, exactly as much power is being lost in the source. Consult the example given in the Table and in Figs. 2 and 3. When 25 watts are dissipated in the resistor  $R_L$ , 25 watts are also dissipated in resistor  $R_s$ . Thus, when the maximum amount of power is transferred, only one-half of the developed power is delivered. Greater *percentages* of developed power can be delivered, and, where the value of  $R_L$  is large compared to the resistor  $R_s$ , practically all the power is delivered to the load. But the *amount* of power then developed and thus delivered is

## IMPEDANCE MATCHING

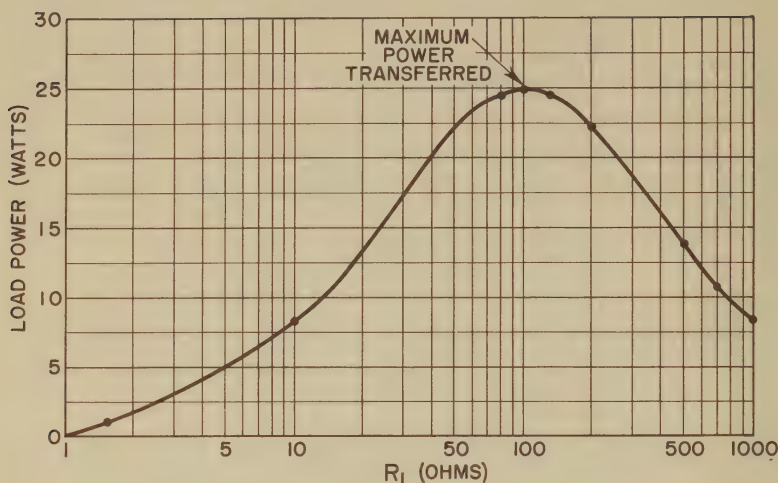


Fig. 3. Graph shows point of maximum power transfer.

small. For example, when  $R_L$  is 10,000 ohms, almost all of the one watt that is developed is delivered to the load, but it is only one watt. Fifty watts are developed and 25 watts are delivered when the values of  $R_s$  and  $R_L$  are equal.

In review then, two important concepts should be understood: first, the greatest amount of power is delivered to a load when the resistance of the source is equal to the resistance of the load, that is, when these resistances match. This is called maximum power transfer. Secondly, when this maximum power transfer condition occurs, half of the generated power is lost in the source and half is dissipated in the load.

Energy can neither be created nor destroyed. The same is true for power. In the example previously illustrated, the battery is able to supply electrical energy by virtue of its conversion from chemical energy. The rate by which this electrical energy is supplied is called electrical power and it is determined by the condition of the external circuit. The electrical energy thus supplied can never be destroyed but only converted by the load to some other form of energy.

As in the case of Fig. 1, the load was a resistance and the only way a resistance can use energy is to convert it into heat. There are many other forms into which the energy expended in the load may be converted. However, in direct current circuits, the source is only concerned with the rate by which energy is leaving (d-c power leaving) not the form of power conversion that takes place

at the load. Because of this, any d-c device may be represented as a resistance in so far as the source is concerned.

For example, an electric motor is a device used to convert electrical power into mechanical power. The power that drives the electric motor of a vacuum cleaner comes from an electrical source. From the viewpoint of this electric source the power it supplies is apparently expended in a resistance, but the power is actually

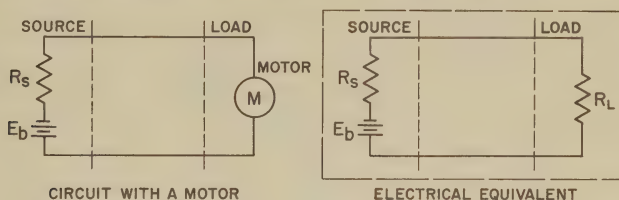


Fig. 4. A power circuit driving a motor, and its electrical equivalent.

used by the motor to perform its required mechanical tasks. This type of conversion, then, is from an electrical to a mechanical form of power. See Fig. 4.

There are a number of possibilities for converting power when a coil of wire is formed about an iron core. The electric power used to move the minute particles of iron in the core, arranging all of their magnetic fields in one direction, represents conversion of electrical power into mechanical power. This is shown in Fig. 5.

As far as the electrical circuit of Fig. 5 is concerned, this conversion also represents dissipated energy in a resistor. In this

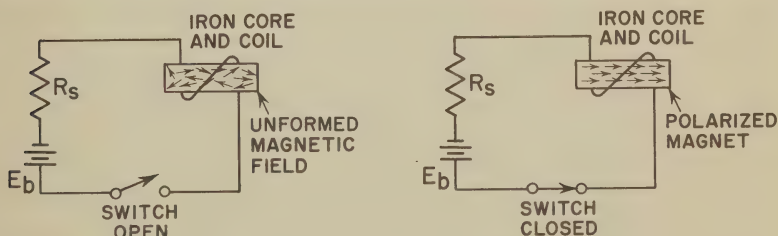


Fig. 5. How conversion of power magnetizes an iron core.

example the power used to move the particles into a magnetic arrangement is called the hysteresis power loss. In other words the amount of hysteresis of a magnetic core is proportional to the amount of power lost when the core is magnetized or demagnetized.

Another form of power conversion occurs in the core as a result

of the flow of current in the core material. The iron acts like the secondary of a transformer and permits current to flow. The flow of current in the core itself against the resistance of the iron causes a power loss. This is shown in Fig. 6.

The flow of current in the core, called an *eddy current*, is undesirable, but it does occur. If the core material is cut into thin laminations (sheets), and each sheet is electrically insulated from

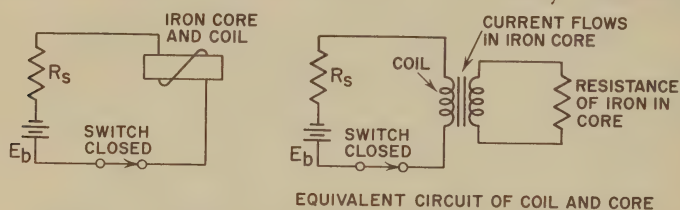


Fig. 6. Conversion of power in the core of an electromagnet.

the others, yet sufficiently close to allow the easy flow of magnetic lines of force, the eddy current is greatly lessened. The flow of eddy current through the resistance of the iron is lost power: It is a direct conversion of power into heat. There is a loss in a laminated iron core when the laminations move as a result of changes in the amount and direction of current flow. Frequently hum has been heard from a transformer while it was operating. This was due to the laminations moving, another example of the conversion of electric power into mechanical power. Again the electric generator "sees" it only as a resistor dissipating power.

The energy given off in magnetic and static fields about a coil of wire is also a power loss. The classic illustration of this type of loss is an antenna in which the electric circuit is arranged to radiate energy. Here, too, the electric circuit "sees" the radiation as a dissipating resistor.

Another interesting result occurs when large voltages are used. The high voltage attracts air particles. As these particles reach the high-voltage region they become charged by losing or gaining electrons, and race from the coil. The energy used to attract and repel the air particles comes from the electric circuit. Some heat and light is produced as well. This *corona loss* is still another example showing conversion of electric energy into radiant energy.

The greatest loss in a coil is due to the actual resistance of the wire itself. This is the *copper loss*.

Observe the number of cases in which electric energy is converted into other forms — motor, hysteresis, eddy, mechanical movement of the laminations, radiation, corona and copper loss. In each case electric power is used. It is converted from its electric form into radiant (light, heat, or electromagnetic) or mechanical



energy. These changes are losses of power. Each one is symbolized in a circuit by a resistor. It is difficult, sometimes, to show these changes. For example, a coil of wire might have only 10-ohms resistance to steady d-c current flow, but when the current is turned on and off periodically, the conversion of power to supply losses due to hysteresis, eddy, lamination movement, radiation, and corona in addition to the copper loss may make the resistance increase several times — from 10 to 100 ohms. All of this change is resistive. All of it is usually shown schematically with one resistor, but it should not be confused with the pure d-c resistance alone.

Resistance, then, symbolizes more than just the resistors in a circuit. It is a means of expressing the loss of power in a circuit — any loss of power. A general statement might be made in this manner: wherever power is used in an electric circuit it is symbolized by resistance. It is important that this concept be fixed in mind. Any nonreversible power conversion in a circuit is resistive. Nonreversible power conversion is never caused by inductance or capacitance.

### 3. Inductance - Capacitance Relationships

While energy may only be converted or dissipated in resistance, other electric devices *store* energy. These are inductors and capacitors, which do not dissipate or convert energy permanently but only store it by temporarily converting it to another form, and then deliver it back to the circuit at another time. When energy is stored in a magnetic field, the electric circuit is called an inductor. *The inductance of a circuit is its ability to store energy in a magnetic field.* As long as energy is being stored in its field, it is an inductance. When the circuit inductance has stored as much supply as it can, additional energy is dissipated by the resistance of the circuit, and the circuit does not have the effect of inductance on the additional energy.

These are very fundamental propositions. Energy is dissipated in resistors. Inductors store energy. Observe the difference. An inductor never, *absolutely never*, dissipates energy. It is an inductor only when it stores energy. This is pictured in Fig. 7. The horizontal axis depicts current flowing through a coil. The vertical axis indicates the amount of magnetism developed. It should be noted, as indicated by the dotted line, that when the current is first caused to flow through the coil, the amount of magnetism developed is very small. Then, as the current is increased, the ability of the core to become more magnetic decreases until any further increase in current does not increase the amount of magnetic field. The core is then said to be saturated. Beyond this point

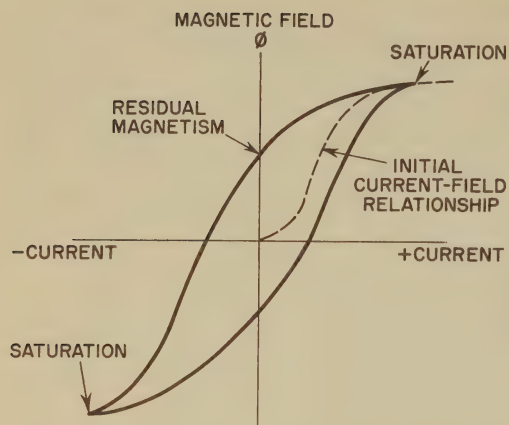


Fig. 7. Inductance and core hysteresis.

the coil no longer behaves like an inductor. It is a resistor only.

As the current is again decreased below saturation, it can be seen that the amount of magnetism does not decrease to zero when the current becomes zero. The amount of magnetism left in the coil when the current is zero is called residual magnetism. As the current is made to flow in the opposite direction, the magnetism decreases to zero. As the current is made greater, the magnetic field becomes saturated in the opposite polarity. The curve is called a hysteresis loop.

All of the electric energy that is stored in an inductor may be recovered. The stored electric energy is neither lost nor converted. If the current commences from zero and rises to a value less than the saturation value, the amount of energy stored in the inductor is:

$$w = \frac{1}{2} LI^2$$

where  $L$  = inductance in henrys;  $I$  = current in amperes.

Once again, it is stressed that electric energy is lost, dissipated, or converted in resistors or resistive components, and stored, not lost, in inductors and capacitors. In inductors the current does not flow immediately upon the application of voltage. The current flow lags the voltage. In capacitors the current leads the voltage. To record this phenomena mathematically, a system of arithmetic (vector notation) is used.

#### 4. Vector Notation

In d-c problems the value of power, voltage, and current can be expressed in units of specific amount. These units define size and

numerical value and give no indication of the direction in which the quantity of the units act. Units which do not have direction noted in them are called *scalar* (amount on a scale, such as a meter scale, distance meter) quantities. Units which describe direction as well as quantity are called *vectors* and may be diagrammed on a vector graph.

Vectors in electric units represent the quantity and direction relationship of currents and voltages, more commonly called phase relationship, in alternating current. The length of the vector represents the amount of the unit involved. The direction (angular placement) of the vector relative to a reference axis indicates the time between the quantity and the reference.

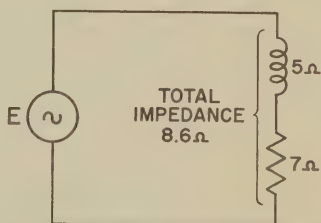
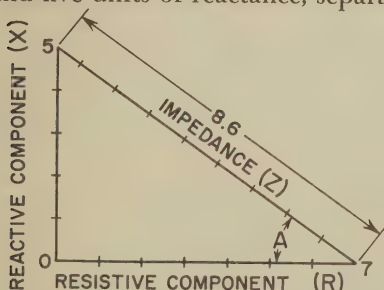
This relationship between size and time is also represented through use of impedance right triangles. In these, the sides represent the size of the reactive and resistive quantities, and the angle separation the time between the two. A right angle triangle is used, the base equaling the resistive component and the vertical leg equaling the reactive (inductive or capacitive) component. Then, the hypotenuse is a measure of the impedance of the two components. Corresponding to the relationship  $Z = \sqrt{R^2 + X^2}$

$Z$  = impedance (hypotenuse)

$R$  = resistance (base)

$X$  = net reactance (altitude).

The impedance triangle of Fig. 8 shows seven units of resistance and five units of reactance, separated by a right angle. This means,



**Fig. 8.** The three components of an impedance triangle, and the computation of total impedance in ohms.

in the series circuit with the same current flowing through both units, that the voltage across the reactive component reaches its maximum value  $90^\circ$  (a quarter of a cycle) earlier than the voltage across the resistive component. The time interval,  $90^\circ$ , may be resolved into time when a frequency is indicated. For example,

for a 60 cycle signal,  $90^\circ$  would be one-fourth of  $16,667 \mu\text{sec}$  ( $90^\circ$  is  $\frac{1}{4}$  of  $360^\circ = 1$  cycle or  $1/60$  of a second, which is  $16,667 \mu\text{sec}$ ) or  $4166 \mu\text{sec}$ . The time was designated *earlier* or leading because an inductor was used. Had a capacitor been used the time would have been *later* or lagging for the reactive component.

In a resistor the current and voltage are always in phase. Since there are no electromagnetic or electrostatic fields to charge, the voltage causes current to flow immediately upon application. For this reason, d-c problems are simpler than a-c.

Returning to a pure inductance (no resistance), the current through lags the voltage across by  $90^\circ$ . Changes in current value and direction occur after changes in voltage. In a pure capacitor the current leads the voltage by  $90^\circ$ ; that is, changes in current level and direction precede changes in voltage by  $90^\circ$ , or the time equivalent of one-quarter of a cycle for the frequency of operation.

In Fig. 8 angle A represents the phase angle between the voltage across the impedance, and the current.

A system of complex numbers is used to represent the relationships described. These are treated as are other more common terms in algebra, with the single exception that an imaginary number is used as an additional factor. Conventional algebraic methods are then adequate.

## 5. The J Operator

The factor  $\sqrt{-1}$  (square root of minus one) is an imaginary number. It is noted in electrical problems as 'j', and sometimes referred to as the 'j operator'. It merely represents the fact that numbers without the letter j in front of them are  $90^\circ$  out of phase with numbers having the letter j.

In Fig. 8 the resistive and reactive components could be written  $7 + j5$ . The usual formulas are:

$$Z \text{ (impedance)} = R + jX_L \text{ (for inductor)} \quad (6)$$

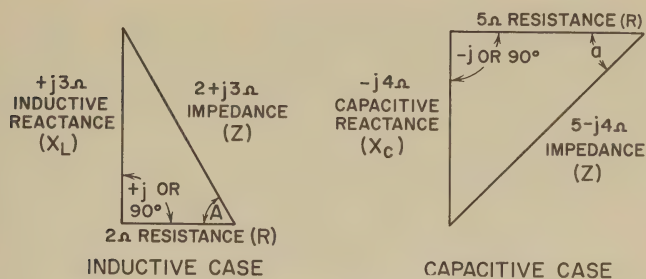
$$Z \text{ (impedance)} = R - jX_c \text{ (for capacitors)} \quad (7)$$

The  $+j$  represents the lagging current angle for inductive reactance and the  $-j$  represents the leading current angle for capacitive reactance. In each case j means  $90^\circ$  out of phase with R.

Complex numbers representing impedance are numbers that have both real numbers for resistive components and imaginary numbers for reactive components. Imaginary numbers are always separated from the real numbers by  $90^\circ$ .

In adding or subtracting complex numbers the usual algebraic forms are used. The real numbers are handled in one section and





**Fig. 9.** Impedance triangles for inductors and capacitors.

the imaginary in another section of the same problem in much the same way as numbers with known factors and numbers with unknown factors in them are handled. Using the values of Fig. 9, the sum or difference of the two numbers would be:

$$\begin{array}{r} 2 + j3 \\ 5 - j4 \\ \hline 7 - j1 \end{array} \quad \text{sum} \quad (8)$$

$$\begin{array}{r} 2 + j3 \\ -(5 - j4) \\ \hline -3 + j7 \end{array} \quad \text{difference} \quad (9)$$

**Illustrative**

**Problem 1:**

Add  $4 + 8$  and  $3 - j6$

**Solution:** The real and imaginary numbers are handled separately.

$$\begin{array}{r} 4 + j8 \\ 3 - j6 \\ \hline \text{Sum} \quad 7 + j2 \end{array}$$

**Illustrative**

**Problem 2:**

Subtract  $-10 - j4$  from  $-4 + j8$

**Solution:** The subtraction process is straightforward if the complex numbers are handled like ordinary binomials. Thus:

$$\begin{array}{r} -4 + j8 \\ -(-10 - j4) \\ \hline \text{Difference} \quad 6 + j12 \end{array}$$

The multiplication and division of complex numbers are similarly handled, with the exception that, since  $j$  equals  $\sqrt{-1}$ ,  $j \times j$  ( $j^2$ ) equals  $-1$ . Thus the square root sign disappears when squaring  $j$ . The use of this is shown in the multiplication of the numbers used above:

$$\begin{array}{r} 5 - j4 \\ 2 + j3 \\ \hline 10 - j8 \\ + j15 - (-1) 12 \\ \hline 10 + j7 + 12 \end{array} = 22 + j7 \quad (10)$$

Illustrative

Problem 3:

Multiply  $3 - j5$  by  $6 + j3$

Solution: The complex numbers are handled like binomials, remembering that  $j^2 = -1$

$$\begin{array}{r} 3 - j5 \\ 6 + j3 \\ \hline 18 - j30 \\ + j9 - j^2 15 \\ \hline \text{Product } 18 - j21 - j^2 15 \end{array}$$

Converting  $j^2$  in the product to  $-1$

$$\begin{array}{r} 18 - j21 - (-1) (15) \\ 18 - j21 + 15 \\ \hline \text{Final product } 33 - j21 \end{array}$$

Illustrative

Problem 4:

Multiply  $7 + j4$  by  $8 + j6$

Solution:

$$\begin{array}{r} 7 + j4 \\ 8 + j6 \\ \hline 56 + j32 \\ + j42 + j^2 24 \\ \hline 56 + j74 + j^2 24 \\ 56 + j74 + (-1) (24) \\ 56 + j74 - 24 \\ \hline \text{Final product } 32 + j74 \end{array}$$

The division of two numbers may be done by rationalizing, that is, by multiplying both numerator and denominator by a factor that will make the denominators a perfect square.

$$\frac{5 - j4}{2 + j3} = \frac{(5 - j4)(2 - j3)}{(2 + j3)(2 - j3)} = \frac{-2 - j23}{13} = -0.15 - j1.8 \quad (11)$$

Illustrative

Problem 5:

Divide  $5 + j6$  by  $3 - j4$

Solution:

Rationalizing

$$\begin{aligned} \frac{(5 + j6)(3 + j4)}{(3 - j4)(3 + j4)} &= \frac{15 + j38 + j^2 24}{9 - j^2 16} \text{ since } j^2 = -1, \text{ then} \\ &= \frac{15 + j38 + (-1) (24)}{9 - (-1) (16)} \\ \text{Final quotient} &= \frac{-9 + j38}{25} \end{aligned}$$

The form of writing impedance as  $R + jX$  is called rectangular because it uses horizontal and vertical coordinates (Figs. 8 and 9).

Impedance may also be written as  $Z/a$ , where the size of the impedance line and its direction is given. To demonstrate this, first solve the value of the impedances for the two cases given in Fig. 9. This may be done by using the Pythagorean theorem. It states that in a right triangle, the length of the hypotenuse (longest side) is equal to the square root of the sum of the squares of the other two sides. That is:

$$2 + j3 = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} = 3.6 \quad (12)$$

In the second problem, note that the sign of the reactive component is assumed positive under the radical. This is because the magnitude of the hypotenuse does not depend on the direction of the legs, only on their magnitude.

$$5 - j4 = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} = 6.4 \quad (13)$$

The figures 3.6 and 6.4 give the size or length of the impedance line. They do not give the angle 'a' of Fig. 8. This angle may be determined from a table of natural tangents. The formula is:

$$\text{the angle 'a' is the angle whose tangent is } \frac{X}{R} \quad (14)$$

For the values in Formulas 12 and 13, the angles' values are easily computed. The angle for 3.6 is taken from a table of natural tangents and is equal to  $3/2$  or  $56^\circ$ . (15)

Similarly, the angle for 6.4 is the angle whose tangent is  $4/5$  or  $39^\circ$ . (16)

Substituting in Formula 12 and 13, we can now write:

$$2 + j3 = 3.6 \angle 56 \quad (17)$$

$$5 - j4 = 6.4 \angle 39 \quad (18)$$

Observe in Formulas 17 and 18 that when the  $j$  had a plus sign in front of it, the symbol to indicate the angle was written  $\angle$ . When the  $j$  had a negative sign the symbol was inverted to  $\angle$ . This nomenclature is convenient but many engineers and technicians still prefer to write  $\angle$  with the plus or minus sign before the number of degrees. Either way is satisfactory.

The size of the impedance determined in Formulas 12 and 13 could also have been obtained by using a table of natural sines. The following formulas, instead of the Pythagorean theorem, would be used:

$$Z = \frac{x}{\sin a} = \frac{R}{\cos a} \quad (19)$$

$$Z = \frac{3}{\sin 56} = \frac{3}{0.83} = 3.6 \text{ or } \frac{2}{\cos 56} = 3.6 \quad (20)$$

And similarly in the second example

$$Z = \frac{4}{\sin 39} = \frac{4}{0.63} = 6.4 \text{ or } \frac{5}{\cos 39} = 6.4 \quad (21)$$

The changing of  $2 + j3$  to  $3.6 \angle 56$  illustrates conversion from rectangular to polar coordinates. To convert from polar to rectangular, the following formula is used:

$$Z \angle a = Z \cos a + jZ \sin a \quad (22A)$$

$$Z \angle a = Z \cos a - jZ \sin a \quad (22B)$$

For the Formulas 17 and 18, the conversion is:

$$\begin{aligned} 3.6 \angle 56 &= 3.6 \cos 56 + j3.6 \sin 56 = 3.6 \times 0.56 + \\ &\quad j3.6 \times 0.83 = 2 + j3 \end{aligned} \quad (23)$$

$$\begin{aligned} 6.4 \angle 39 &= 6.4 \cos 39 - j6.4 \sin 39 = 6.4 \times 0.78 - \\ &\quad j6.4 \times 0.63 = 5 - j4. \end{aligned} \quad (24)$$

#### Illustrative

Problem 6: Find the resistance and reactance of a series circuit having an impedance of 2690 ohms and a leading phase angle of  $-21.8$  degrees.

Solution: Expressed in polar form, the terms of the problem are:

$$Z = 2690 \angle 21.8^\circ \text{ ohms}$$

These values may now be substituted in (Eq. 22b):

$$Z = 2690 \cos 21.8^\circ - j2690 \sin 21.8^\circ$$

$$Z = 2690 (0.9285) - j2690 (0.3714)$$

$$Z = 2498 - j999.1 \text{ ohms}$$

When rounded back to one less significant figure, this result becomes:

$$Z = 2500 - j1000 \text{ ohms}$$

The addition or subtraction of complex numbers is awkward if the polar forms are used, because the results do not appear directly in polar form. For these operations the rectangular form is much more convenient. However, polar forms allow multiplication and division to be handled simply.

To use the polar form of numbers for multiplication, multiply the impedances and algebraically add the angles. Remember that  $\angle$  is a plus angle and  $\angle$  is a negative angle. For example:

$$3.6 \angle 56 \times 6.4 \angle 39 = 3.6 \times 6.4 \angle 56 + 39 = 23 \angle 95 \quad (25)$$



Division of polar forms is accomplished by dividing the numbers and subtracting the angles; the phase angle of the divisor is the subtrahend.

$$\frac{3.6 \angle 56}{6.4 \angle 39} = \frac{3.6 \angle 56 - (-39)}{6.4} = \frac{3.6 \angle 56 + 39}{6.4} = 0.56 \angle 95 \quad (26)$$

## 6. Impedance of Resonant Circuits

Resonant circuits may have the inductive and capacitive portions of the circuit either in series or in parallel. When in series, it is rightfully called a series resonant circuit. When in parallel, it is called by different names: parallel, shunt, or antiresonant.

The condition of resonance for a series circuit occurs at that frequency where the reactances of the capacitor and of the inductor are equal. This is usually written:

$$X_c = X_L \quad (27)$$

The impedance of the resonant circuit (series) is expressed in the following manner:

$$Z = R + j(X_L - X_c) = R \quad (28)$$

The formula is reduced to  $R$  alone because the reactances were equal and of opposite signs and therefore cancelled each other out.

The frequency at which the circuit is resonant can be computed from Formula 27 in the following manner:

$$X_c = X_L \quad (29)$$

$$\frac{1}{2\pi f C} = 2\pi f L \quad (30)$$

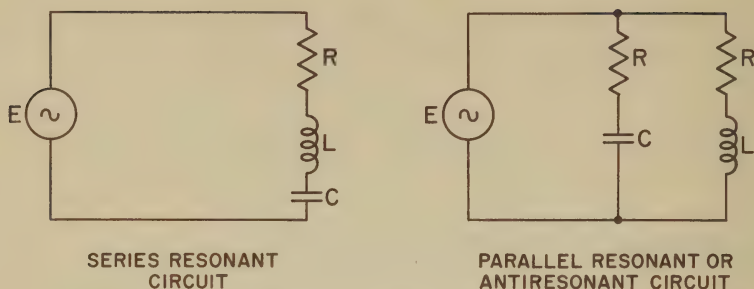
where  $L$  is in henrys and  $C$  in farads. By transferring the factors across the equation sign, we obtain:

$$f^2 = \frac{1}{(2\pi)^2 LC} \quad (31)$$

$$f = \frac{1}{2\pi \sqrt{LC}} \quad (32)$$

where  $f$  is frequency in cycles per second.

In the series circuit of Fig. 10, the same current flows in all three components. The voltage across the resistor is in phase with the current, as it always is. The voltage across  $L$  leads the current by



**Fig. 10.** Resonant circuits.

$90^\circ$ . Across the capacitor  $C$  it lags the current by  $90^\circ$ . Therefore, as the voltages across the inductor are leading by  $90^\circ$  and across the capacitor are lagging by  $90^\circ$ , the voltages are  $180^\circ$  out of phase with each other. There is complete cancellation of the voltages across the reactive elements of the circuit. Only the voltage across the resistor is left. This is the voltage which limits the current flow, and as a result, the voltage across the resistor is equal to the source voltage. The inductor and capacitor do not limit current flow. The voltages across  $L$  and  $C$  drive current through their impedance. However, as the power delivered to the reactive elements is all returned by them, no power is dissipated in either one. Power is consumed or converted into heat in the resistor only.

The ratio of the energy stored in an inductor or capacitor to the energy used in the resistor is a measure of how well the unit acts as a reactive element. This may be written for an inductor:

$$\frac{I^2 X_t}{I^2 R_t} = \frac{X_t}{R_t} = Q \quad (33)$$

where  $X_t$  and  $R_t$  are the total inductive reactance and resistance of the unit being considered. The formula shows that  $Q$  is a ratio of the inductive reactance to the resistive ohms. These resistor ohms include, of course, all of the losses previously discussed: hysteresis, eddy, copper, corona, radiation, and mechanical.

Referring back to Fig. 9, the impedance triangle for the inductive case  $2 + j3$ , the  $Q$  is:

$$Q = \frac{X}{R} = \frac{3}{2} = 1.5 \text{ or the tangent of the angle } A \text{ or } 56^\circ. \quad (34)$$

Similarly for the capacitive case of Fig. 9:

$$Q = \frac{X}{R} = \frac{4}{5} = 0.8 \text{ or the tangent of the angle } A \text{ or } 39^\circ \quad (35)$$

Capacitors normally have almost no resistive components and as a result practically all of the resistance in a series-resonant circuit is in the inductor. The  $Q$  of the resonant circuit, therefore, determines the current flow. The current flow determines the voltage across the reactive components. When  $R$  is small the voltage across  $C$  becomes large. The voltage across  $C$  can be greater than the supply voltage. This is called voltage gain ( $VG$ ). Then:

$$E_c = VG = \frac{X_c}{R} = Q \quad (36)$$

where  $R$  is total series  $R$ .

Illustrative

Problem 7: Find the  $Q$  of a predominantly resistive circuit in which the impedance is  $400 + 150j$ . What is the phase angle in this circuit?

Solution: The resistance of this circuit is 400 ohms and the inductive reactance is 150 ohms. Substituting in equation 33:

$$Q = \frac{X_L}{R} = \frac{150}{400} = 0.37$$

(Note that  $Q$  has no units since it is a ratio of two ohmic oppositions in which the units cancel.)

$Q$  is the tangent of the phase angle  $\Phi$ , so that

$$\tan \Phi = 0.37$$

$$\Phi = 20.3^\circ$$

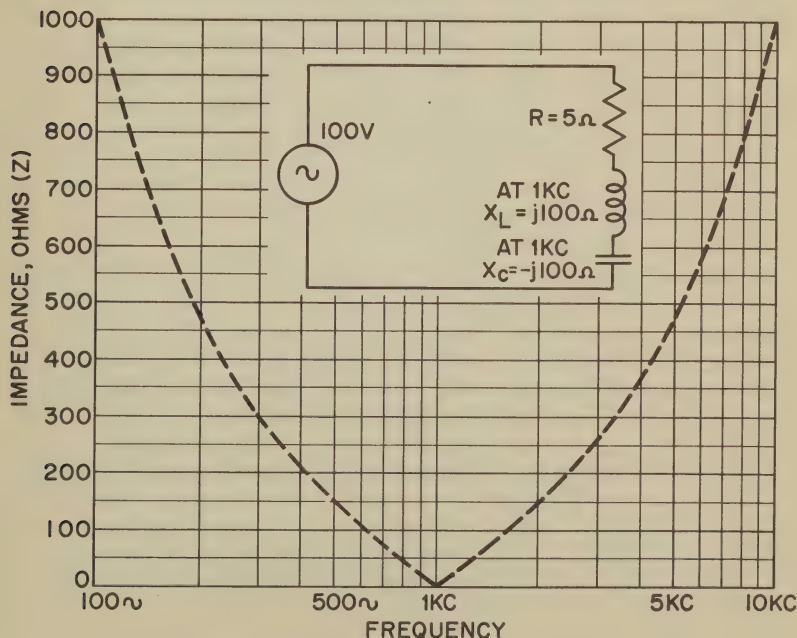


Fig. 11. Impedance curve in series-resonant circuit.

At other than the resonant frequency, the current through a series-resonant circuit is smaller. Not only does the resistance limit the current flow at nonresonance, but it also limits the remaining reactance ohmic value after the difference between the two reactances has been taken. This is shown in Fig. 11 with a table of the values computed from Formula 28. Remember that the circuit is resonant at only one frequency, which is arbitrarily selected at 1000 cycles. At this frequency the reactances are 100 ohms, and the resistance five ohms. This is a reasonably high  $Q$  circuit.

TABLE 2

cps	$R^2$	$X_L$	$X_c$	$X_L - X_c$	$X^2$	$X^2 + R^2$	$\sqrt{R^2 + X^2}$	I amps
100	25	10	1000	990	980000	980025	990	0.101
200	25	20	500	480	230000	230025	480	0.208
300	25	30	333	303	92000	92025	303	0.330
500	25	50	200	150	22500	22525	150	0.667
800	25	80	125	45	2020	2045	45	2.22
900	25	90	111	21	441	466	21	4.76
1000	25	100	100	0	0	25	5	20.00
1110	25	111	90	21	441	466	21	4.76
1250	25	125	80	45	2020	2045	45	2.22
2000	25	200	50	150	22500	22525	150	0.667
3330	25	333	30	303	92000	92025	303	0.330
5000	25	500	20	480	230000	230025	480	0.208
10000	25	1000	10	990	980000	980025	990	0.101

Below resonance,  $X_c$  predominates and produces a leading current which is smaller than that at resonance. Above resonance,  $X_L$  predominates and causes a lagging current which is again smaller than that at resonance.

At resonance  $X_L = -X_c$  and the reactances therefore cancel each other out. The circuit impedance reduces to a pure resistance and is at a minimum. The current is therefore at a maximum at resonance and is in phase with the voltage.

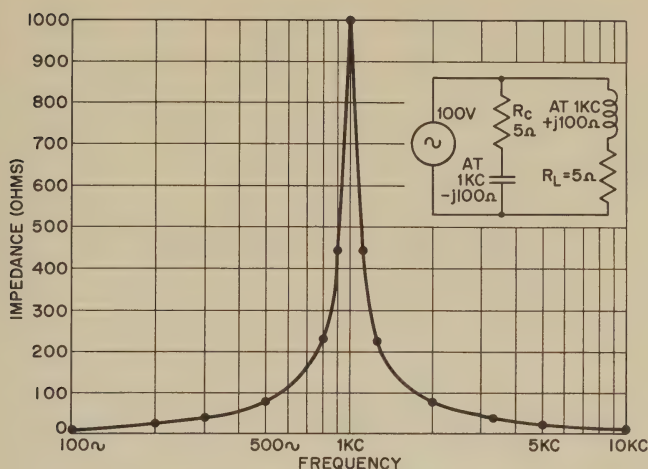
It is interesting to note the voltage across the reactive elements at resonance for the circuit and data of Fig. 12:

$$E_c = E_L = I X = 20 \text{ amperes} \times 100 \text{ ohms} = 2000 \text{ volts} \quad (37)$$

This demonstrates that the voltage across the reactive elements at resonance can be and often is many times the source voltage.

Returning to the parallel-resonant circuit and using the same description of resonance as was used in the series-resonant circuit, we see the variation of impedance versus frequency in Fig. 12.





**Fig. 12.** Impedance curve in a parallel-resonant circuit.

The  $Z_c$  computations are merely the reversed order of the  $Z_L$  computations. The formula for parallel impedances is the same as for parallel resistors, except that it is easier to convert forms from the rectangular to the polar system to make the final column.

Figure 12 shows that the impedance at resonance is very large and therefore the line current is very small.

The definition of resonance taken from the series resonant case applies in Fig. 12. However, it must be remembered that in parallel resonance the line current should be minimum.

## Review Questions

1. Define maximum power transfer.
2. Under what condition is the greatest amount of power transferred?
3. When the greatest amount of power is transferred, what percent of the total power is delivered to the load?
4. List six types of power losses.
5. Describe hysteresis, eddy current, corona, and copper losses.
6. Do pure inductors or capacitors dissipate power?
7. What occurs when a core becomes saturated?
8. What is the phase angle separation between resistive and reactive components?
9. Give the rectangular and polar forms of the equations for impedance.
10. What purpose does the  $j$  operator serve?
11. Express the impedance of a 10-henry choke and a 10- $\mu$ f capacitor in rectangular and polar forms at a) 10 cycles; b) 100 cycles.
12. Is the line current supplied from a source maximum at resonance for a series or parallel resonant circuit?
13. What does  $Q$  represent?

## Chapter 2

### IMPEDANCE-MATCHING DEVICES

#### 7. Power Comparison in Impedance Matching

In this chapter consideration will be given to some of the devices commonly used to match the source to the load. In the d-c example discussed previously, the load was adjusted until an adequate match was made between the source and load. This is satisfactory procedure when considering a theoretical problem or a new design. In practice, however, components cannot always be readily adjusted to meet maximum power transfer requirements. Instead, some device must be connected between the source and the load to obtain proper matching.

Transformers, tube circuitry (especially the cathode-follower type of circuit), resistive networks, and reactive networks are some of the devices used, and each of these will be described separately. In the descriptions, it is helpful to compare the power delivered to the load with and without the matching device. In some cases a percentage figure is used, although many times it becomes meaningless because of its minuteness.

If the circuit of Fig. 13 is used, it is readily recognized that the power delivered is

$$w = I^2 R \quad (38)$$

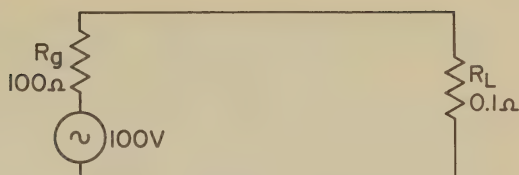
where the current flowing is:

$$I = \frac{E}{R} = \frac{100}{100.1} = 1 \text{ ampere} \quad (39)$$

and therefore the power delivered to the load resistor is:

$$w = I^2 \times 0.1 = 0.1 \text{ watt}$$

**Fig. 13.** Circuit which delivers a small amount of power to the load.



Voltages and currents will always be taken as rms values.

The power delivered, had the resistor been 100 ohms, would have been 25 watts. This means that only 0.1 watt of a possible 25 watts is delivered which is 0.4%. This percentage is small. For purposes of comparison, consider the audio amplifier in a radio set where the voltage increase is about a thousand fold.

To transform the ratios of power into quantities that can be easily understood, consider the following table which gives a shorthand for large numbers.

**TABLE 3**

Number	Expressed as a power of 10
.001	$1 \times 10^{-3}$
.01	$1 \times 10^{-2}$
0.1	$1 \times 10^{-1}$
1.	$1 \times 10^0$
10.	$1 \times 10^1$
100.	$1 \times 10^2$
1000.	$1 \times 10^3$
10000.	$1 \times 10^4$
100000.	$1 \times 10^5$
1000000.	$1 \times 10^6$

The numbers to the right and above the 10 in the power-of-10 column are called exponents. These indicate how many times 10 had to be multiplied by itself to form the number in the left-hand column.

#### Illustrative

**Problem 8:** Express the following in exponential form: 33,000, 610, 86,300,000, 0.003, 0.0000017.

**Solution:**

$$\begin{aligned}
 33,000 &= 3.3 \times 10^4 \\
 610 &= 6.1 \times 10^2 \\
 86,300,000 &= 8.63 \times 10^7 \\
 .003 &= 3 \times 10^{-3} \\
 .0000017 &= 1.7 \times 10^{-6}
 \end{aligned}$$

Note: It is customary to express all numbers having more than one significant figure in a form which permits only one digit preceding the decimal point. This convention reduces the awkwardness of handling certain types of computations.

This table has a value beyond showing the powers of 10. Let us select any number between two steps in the table. For example, between 10 and 100; 20. Obviously 20 must be a number between  $10^1$  and  $10^2$ . To describe it precisely one must use a decimal between 1 and 2. A logarithmic table, similar to the next table, indicates how big the fraction is.

Number	Logarithm	Number	Logarithm
10	.00	55	.74
15	.18	60	.78
20	.30	65	.81
25	.40	70	.85
30	.48	75	.88
35	.54	80	.90
40	.60	85	.93
45	.65	90	.95
50	.70	95	.98

This table tells us that if the first part of the number we are seeking is 2 and the following part is 0, the logarithm is 0.30. This 0.30 is the fraction that is to be placed after the whole number we found in the power-of-10 table. Then for the number 20, with an exponent of 1 and a fractional part of 0.30, the full exponent would be  $10^{1.30}$ . We would similarly find that 40 has 10 raised to the 1.60 power, that is  $40 = 10^{1.60}$ .

We now have another means of expressing any number by determining the exponent and adding to the exponent a fractional exponent from a logarithm table. Thus, 20 is  $10^{1.3}$ , 200 is  $10^{2.3}$ , 2000 is  $10^{3.3}$ , etc.

#### Illustrative

Problem 9: Express the following in exponential form using logarithmic exponents: 75, 8000, 550.

Solution:

$$\begin{aligned} 75 &= 10^{1.88} \\ 8000 &= 10^{3.9} \\ 550 &= 10^{2.7} \end{aligned}$$

This gives us a means of expressing numbers. If instead of writing the 10 each time, we were to write only the exponent, it would be more convenient. This was recognized some years ago and at a conference of scientific people at London the name of the exponent was changed to *Bel*. For many purposes the Bel unit was too large.



It was sub-divided by 10, and the new unit retermed "decibels". Thus, the number 20 is 1.3 bels or 13 decibels.

Illustrative

Problem 10: Convert the numbers of illustrative problem #9 to decibels (db).

Solution:

$$\begin{aligned} 75 &= 18.8 \text{ decibels} \\ 8000 &= 39 \text{ decibels} \\ 550 &= 27 \text{ decibels} \end{aligned}$$

Decibels are used only to express ratios. For the example given in Fig. 13, the ratio of 0.4%, or .004 of the power that could have been transferred was delivered to the tiny resistor. From the two tables given above it is seen the number .004 can be expressed as  $10^{-3.6}$  or as 36 db below the power that could have been delivered. By observation one can determine whether there is a decibel loss or gain. In the example above, it is obvious that the 36 db represents a loss. A decibel loss has a minus sign associated with it.

Decibels, then, are ratios that make it easier to express, and therefore to understand, great differences in power. In the above example, it is easier to say 36 db than .004, or even 0.4%. It becomes even more convenient when we compare the hearing limits of the ear. The ratio between the smallest and the largest sound that a normal ear can hear at any frequency is about one to 100,000,000,000,000, a number difficult to comprehend. To express it in decibels (140 db) is much easier.

Illustrative

Problem 11: A circuit such as Fig. 14 has the following values:  $R_G = 75 \Omega$ ,  $R_L = 25 \Omega$ ,  $E = 10$  volts. Find the power loss in db.

Solution:

(1) Find I

$$I = \frac{E}{R_{\text{total}}} = \frac{10}{100} = 0.1 \text{ ampere}$$

(2) Find total power consumption ( $P_1$ ):

$$P_1 = E \times I = 10 \times 0.1 = 1 \text{ watt}$$

(3) Find power consumption of  $R_L$  ( $P_2$ ):

$$P_2 = I^2 R_L = (0.1)^2 \times 25 = 0.25 \text{ watt}$$

(4) Then

$$\text{No. of db} = 10 \log_{10} \frac{P_1}{P_2} = 10 \log_{10} \frac{1}{0.25} = 10 \log_{10} 4 = 6$$

The power loss is 6 db.

The mathematical expression for deriving the decibel gain (+db) or loss (-db) of any two power levels is:

$$\text{No. of db} = 10 \log_{10} \frac{P_1}{P_2} \quad (40)$$

where  $\log_{10}$  = the logarithm to the base 10,  $P_1/P_2$  = the ratio of the two power levels.

If we wish to express the db gain or loss in terms of voltage or current, we need but substitute in the above formula  $E^2/R$  or  $I^2R$  respectively. Then:

$$\text{No. of db} = 10 \log_{10} \frac{\frac{E_1^2}{R}}{\frac{E_2^2}{R}} = 10 \log_{10} \frac{E_1^2}{E_2^2} \quad (41)$$

$$\text{No. of db} = 10 \log_{10} \left( \frac{E_1}{E_2} \right)^2 = 2 \times 10 \log_{10} \frac{E_1}{E_2} \quad (42)$$

$$\text{No. of db} = 10 \log_{10} \frac{I_1^2 R}{I_2^2 R} = 10 \log_{10} \left( \frac{I_1}{I_2} \right)^2 = 20 \log_{10} \frac{I_1}{I_2} \quad (43)$$

Formulas 42 and 43, using voltage and current ratios to determine decibel gain or loss, can be used only if the two resistances or impedances are of like value. If they are different these formulas must be changed to read:

$$\text{No. of db} = 20 \log_{10} \frac{E_1 \sqrt{Z_2}}{E_2 \sqrt{Z_1}}$$

$$\text{No. of db} = 20 \log_{10} \frac{I_1 \sqrt{Z_1}}{I_2 \sqrt{Z_2}}$$

### 8. Transformer as Impedance-Matching Device

The most commonly used impedance-matching device is the transformer. It permits us to connect a source of one impedance to a load of another impedance without suffering the losses of an impedance mismatch. Such a connection effects an improvement in the amount of power transferrable between devices of different impedance levels.

It is assumed that the reader has a working knowledge of the action of a transformer. In review, however, a transformer may be defined as any device used to couple energy inductively from

one circuit to another. The current flow through a primary winding establishes a magnetic field about another, a secondary, winding. The two windings are in a common magnetic field. The energy of the primary winding is transferred by the magnetic field to the secondary winding, where a current is developed from the action of the magnetic field on the secondary winding itself. The magnetic field's coupling of the two windings is the means by which power in the primary winding is transferred to the secondary winding. The transformer establishes the amount of energy transfer from the primary to the secondary, and also establishes the levels of primary and secondary impedances at the intended operating frequencies.

**Mutual Impedance.** A simple form of transformer action is shown in Fig. 14 where two windings are so connected in series that the

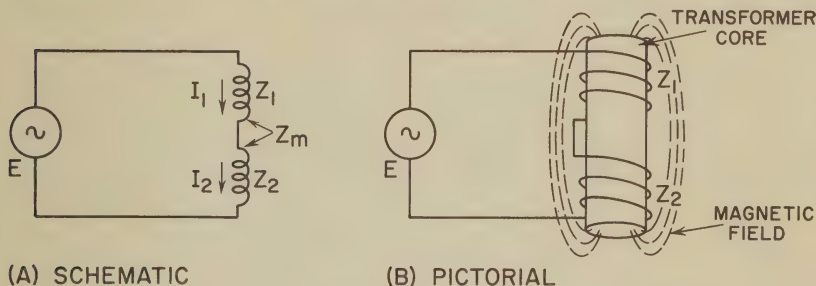


Fig 14. Series-aiding windings.

magnetic fields of the two windings add to each other. The impedance of the windings, and the currents through the windings, are noted with the subscripts <sub>1</sub> and <sub>2</sub>. The mutual impedance is subscripted <sub>m</sub>; mutual impedance is the impedance created by the windings' proximity to one another. It is preferable to use notations like these rather than "primary" and "secondary"; the flow of power is sometimes reversed and then the use of "primary" and "secondary" is confusing.

The voltages around the circuit of Fig. 14 are:

$$E = I_1 Z_1 + I_2 Z_2 + I_1 Z_m + I_2 Z_m \quad (44)$$

That is, the voltage  $E$  is the sum of the voltages acting in the two windings. The four voltages shown above are:

- (1) the voltage drops across the impedance of winding 1,
- (2) the voltage drop across the mutual impedance appearing in winding 1, caused by the flow of current in winding 2,
- (3) the voltage drop across the impedance of winding 2,

(4) the voltage drop across winding 2 to overcome the mutual impedance caused by the flow of the current in winding 1.

Where the two windings of Fig. 14 are equal in the number of turns and therefore in impedance, the above formula reduces to:

$$E = 2(I_1 Z_1 + I_1 Z_m) = I_1 Z_1 + I_2 Z_2 + 2I_1 Z_m \quad (45)$$

Note that  $I_1 = I_2$  because the same current flows in the series circuit components. Since the impedances are the same, the voltages acting in the circuit may be written either way. The second form of Formula 45 is more commonly used and expresses the voltages acting in each winding separately plus the mutual impedance.

**Parallel Connection.** If the two windings are connected in parallel the circuit appears as shown in Fig. 15.

The voltages acting in this circuit may be analyzed in the same manner as those in the previous figure, using the series voltage

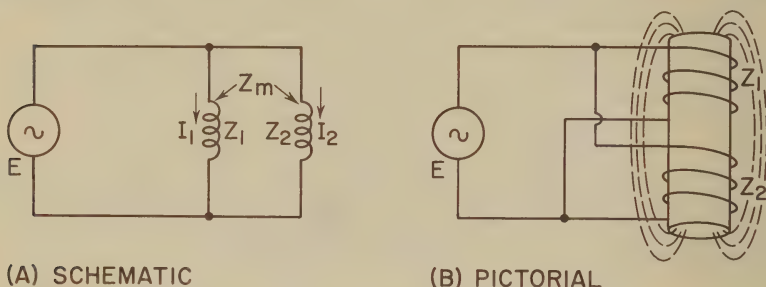


Fig. 15. Parallel-aiding windings.

law of Kirchhoff if each branch or winding is considered separately. The formula may be written:

$$E = I_1 Z_1 + I_2 Z_m = I_2 Z_2 + I_1 Z_m \quad (46)$$

The formula states that the voltages contributed by the self impedance plus the mutual impedance of each branch equals the total voltage across the branch. The current in each branch may be computed from the following formulas:

$$I_1 = \frac{E(Z_2 - Z_m)}{Z_1 Z_2 - Z_m^2} \quad (47)$$

$$I_2 = \frac{E(Z_1 - Z_m)}{Z_1 Z_2 - Z_m^2} \quad (48)$$

The total current, which is the current flowing from the source, is the sum of these two formulas:



$$I_t = I_1 + I_2 = \frac{E(Z_1 + Z_2 - 2Z_m)}{Z_1 Z_2 - Z_m^2} \quad (49)$$

From this the total impedance of the load may be determined:

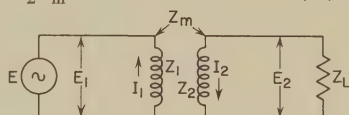
$$Z_L = \frac{E}{I_t} = \frac{Z_1 Z_2 - Z_m^2}{Z_1 + Z_2 - 2Z_m} \quad (50)$$

**Two Windings.** From the discussion the more common case naturally follows, that of a transformer with two separate windings connecting a source and load impedance. This is shown in Fig. 16.

In this circuit the series voltages are:

$$E_1 = I_1 Z_1 + I_2 Z_m \quad (51)$$

**Fig. 16.** Schematic diagram of a two-winding transformer.



If the windings are equal in number of turns, the voltage of the first winding equals the voltage in the second winding due to the self and mutual impedances.

Similarly, the second winding voltages total zero by applying Kirchhoff's voltage law:

$$I_2(Z_2 + Z_L) + I_1 Z_m = 0 \quad (52)$$

By combining Formulas 51 and 52 the currents in the windings are found to be:

$$I_1 = \frac{E_1(Z_L + Z_2)}{Z_1(Z_L + Z_2) - Z_m^2} \quad (53)$$

$$I_2 = \frac{-E_1 Z_m}{Z_1(Z_L + Z_2) - Z_m^2} \quad (54)$$

The minus sign in front of  $E_1$  means that the transformer causes phase inversion.

If it were assumed that  $Z_1$  and  $Z_2$  had no coupling loss (in a perfect transformer this would be the case) then:

$$Z_m = \sqrt{Z_1 Z_2} \quad (55)$$

and the current Formulas 53 and 54 would reduce to

$$I_1 = \frac{E_1(Z_L + Z_2)}{Z_1 Z_L} \quad (56)$$

$$-I_2 = \frac{E_1 \sqrt{Z_2}}{Z_L \sqrt{Z_1}} \quad (57)$$

$$\frac{-I_2}{I_1} = \frac{\sqrt{Z_1 Z_2}}{Z_L + Z_2} \quad (58)$$

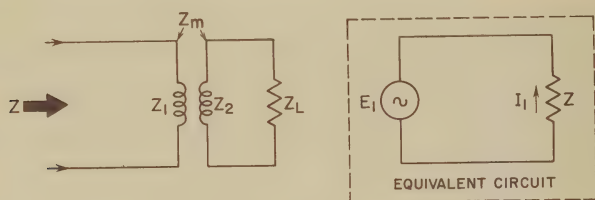
If now the magnitude of  $Z_2$  is much greater than the magnitude of  $Z_L$ , then one obtains

$$\frac{-I_2}{I_1} = \sqrt{\frac{Z_1}{Z_2}}, \quad \frac{I_2^2}{I_1^2} = \frac{Z_1}{Z_2} \quad (59)$$

Formula 59 says that the ratio of the currents in first and second winding is inversely proportional to the ratio of the square root of the self impedance of the windings. The negative sign of  $I_2$  indicates phase inversion.

The impedance looking through the first winding to the load is indicated as  $Z$  and is shown in Fig. 17.

The impedance  $Z$  is a single impedance which, when placed in the primary circuit, represents the combined effect of the load im-



**Fig. 17.** Diagram of load impedance as seen through a transformer, and an equivalent circuit.

pedance and its transformation. In other words  $Z$  is a single impedance which would draw the same current  $I_1$  that flows in the presence of the transformer and load. One can write

$$E_1 = ZI_1$$

Since Formula 56 may be written in the form

$$E_1 = \frac{Z_1 Z_L}{Z_L + Z_2} I_1$$

it is seen that

$$Z = \frac{Z_1 Z_L}{Z_L + Z_2}$$

If, as in Formula 59, it is assumed that the magnitude of  $Z_2$  is much greater than the magnitude of  $Z_L$ , then

$$Z = \frac{Z_1 Z_L}{Z_2} \quad (60)$$

### Illustrative

**Problem 12:** Find the impedance of the load as seen through the windings of the transformer if, using Fig. 17,  $Z_1 = 100 \Omega$ ,  $Z_2 = 200 \Omega$ ,  $Z_L = 400 \Omega$ .

**Solution:** Since the formulas used require that  $Z_2$  be large in magnitude compared with  $Z_L$  while  $Z_1$  and  $Z_2$  usually have the same order of magnitude, the numbers in the problem are improper. Fortunately, if the values of  $Z_1$  and  $Z_2$  are made large with their ratio kept fixed, the solution will not change. For example, use  $Z_1 = 10,000 \Omega$  and  $Z_2 = 20,000 \Omega$ .

$$Z = \frac{Z_1 Z_L}{Z_2} = \frac{100 \times 400}{200} = 200 \Omega$$

The voltages across the windings are in the ratio:

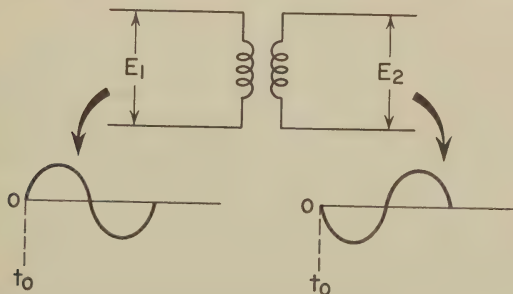
$$\frac{E_1}{E_2} = \frac{I_1 Z}{I_2 Z_L} = \frac{I_1}{I_2} \times \frac{Z}{Z_L} \quad (61)$$

and by substituting:

$$\frac{E_1}{E_2} = -\sqrt{\frac{Z_2}{Z_1}} \times \frac{\frac{Z_1 Z_L}{Z_2}}{\frac{Z_L}{Z_1}} = -\sqrt{\frac{Z_1}{Z_2}} \quad (62)$$

Here is a statement that relates the voltage across each of the windings with their impedances: *the voltages are directly proportional to the square root of the self impedances of the windings.* The negative sign again indicates that the voltage is in phase, inverted by crossing from one winding to the other. This is diagrammed in Fig. 18. It has been mentioned several times in previous formulas and now will be discussed in detail.

The phase inversion that occurs when a signal passes through a transformer may be explained in the following manner. At the



**Fig. 18.** Representation of the phase inversion of voltage.

time  $t_0$ , the start of the  $E_1$  voltage, the current that flows in the  $Z_1$  winding starts  $90^\circ$  later than the voltage. The  $Z_1$  current causes a magnetic field about the core to form. This field in turn cuts the wires of the  $Z_2$  winding, inducing a current that  $90^\circ$  later

establishes the  $E_2$  voltage. The  $E_2$  voltage is then  $180^\circ$  (or out of phase) with the  $E_1$  voltage. This is a practical nonrigorous explanation of phase inversion adequate for our purposes.

The voltage induced in the windings is directly proportional to the number of turns. This may be reduced to a formula written:

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} = \sqrt{\frac{Z_1}{Z_2}} \quad (63)$$

where the minus sign before the last term has been omitted.

Formula 63 combines with Formula 62 to permit the following statements. Voltages of each winding are (a) directly proportional to the number of turns in the winding and (b) directly proportional to the square root of the self impedance of the windings.

#### Illustrative

Problem 13: Given, using Fig. 17,  $Z_1 = 40,000 \Omega$ ,  $Z_2 = 30,000 \Omega$ ,  $Z_L = 100 \Omega$

Find: (a) the impedance of the load as seen through the windings of the transformer.

(b) the voltage ratio between primary ( $Z_1$ ) and secondary ( $Z_2$ ).

(c) the turns ratio

Solution:

$$(a) \quad Z = \frac{Z_1 Z_L}{Z_2} = \frac{400 \times 100}{300} = 133 \Omega$$

$$(b) \quad \text{voltage ratio} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{400}{300}} = 1.15$$

The voltage ratio is 1.5 : 1

$$(c) \quad \frac{E_1}{E_2} = \frac{T_1}{T_2} = \sqrt{\frac{Z_1}{Z_2}}$$

The formula tells us that voltage ratio and turns ratio are the same. This means the turns ratio of the coil should be 1.5 : 1.

We now have four important rules about transformers that are needed to solve impedance matching problems:

1. Mutual impedance equals the square root of the product of self impedances of the windings. (See Formula 55.)

2. The currents in the windings are inversely proportional to the square root of the self impedances of the windings. (See Formula 59.)

3. The voltages across the windings are directly proportional to the square root of the self impedances of the windings. (See Formula 62.)

4. The voltages across the windings are directly proportional to the number of turns or the square root of the impedances. (See Formula 63.)



**Autotransformers.** These rules may be applied to an autotransformer such as the one shown in Fig. 19.

In Fig. 19, the voltages about the circuit are:

$$E = I_2 Z_t = I_2 Z_2 + (I_2 - I_L) Z_m + I_L Z_L \quad (64)$$

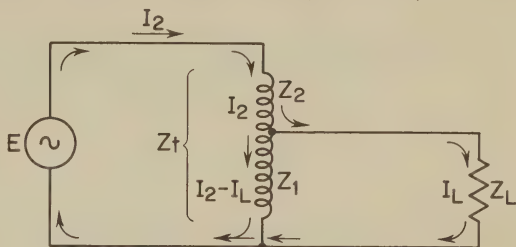
Similarly, the currents involved are:

$$(I_2 - I_L) Z_1 + I_2 Z_m = I_L Z_L \quad (65)$$

$$I_2 = \frac{E (Z_1 + Z_m)}{(Z_1 + Z_L) (Z_2 + Z_m) + (Z_1 + Z_m) (Z_L - Z_m)} \quad (66)$$

$$I_L = \frac{E (Z_1 + Z_m)}{(Z_1 + Z_L) (Z_2 + Z_m) + (Z_1 + Z_m) (Z_L - Z_m)} \quad (67)$$

**Fig. 19.** Series-aiding autotransformer.



From the Formula of  $I_2$ , the total impedance can be determined:

$$Z_t = \frac{E}{I_2} = \frac{(Z_1 + Z_L) (Z_2 + Z_m) + (Z_1 + Z_m) (Z_L - Z_m)}{Z_1 + Z_L} \quad (68)$$

If the transformer is assumed to be perfect so there is no loss, then  $Z_1$  and  $Z_2$  are very large in magnitude; and further  $Z_m = \sqrt{Z_1 Z_2}$ , then:

$$Z_t = \frac{(Z_1 + Z_2 + 2Z_m) Z_L}{Z_1} = \frac{(\sqrt{Z_1} + \sqrt{Z_2})^2 Z_L}{\sqrt{Z_1}} \quad (69)$$

if  $Z_L$  is neglected in the denominator.

Combining this with Formula 63 which stated that the number of turns is directly proportional to the square root of the impedance:

$$Z_t = \frac{(T_1 + T_2)^2}{T_1} Z_L \quad (70)$$

This formula may be resolved into a rule: the impedance looking into an autotransformer is equal to the square of the ratio of the entire winding turns to the partial winding turns, times the impedance of the load connected to the partial winding.

## Illustrative

Problem 14: Fig. 19,  $T_1 = 1000$  turns,  $T_2 = 2000$  turns,  $Z_L = 200 \Omega$

Find: the total impedance  $Z_t$

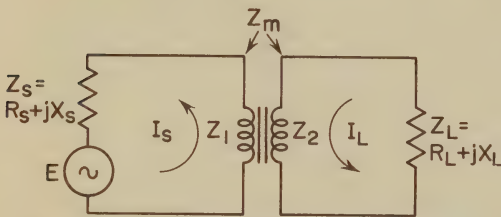
Solution: Using Formula 70 we get

$$Z_t = \left( \frac{T_1 + T_2}{T_1} \right)^2 Z_L = \left( \frac{1000 + 2000}{1000} \right)^2 \times 200 = 1800 \Omega$$

In the above discussion of transformer and autotransformer fundamentals, it was assumed that a perfect transformer was available. Where there is such a transformer, it would not use power from the circuit supply. If this hypothetical perfect transformer were placed in a circuit between two unequal impedances, a transformer ratio could be determined so that maximum power could be transferred.

A more realistic example of power transfer may be illustrated with the values of the circuit shown in Fig. 20.

In this circuit we can develop the load current, the maximum



**Fig. 20.** Schematic of a transformer matching two impedances.

load current, and the conditions under which the greatest amount of power can be transferred.

The voltages in the source series circuit are:

$$E = I_s(Z_1 + Z_s) + I_L Z_m \quad (71)$$

The voltages in the secondary circuit add up to zero by Kirchhoff's Law:

$$I_L Z_2 + I_s Z_m + I_L Z_L = 0 \quad (72)$$

The current in the load is then computed from these formulas:

$$I_L = \frac{-E Z_m}{(Z_1 + Z_s)(Z_L + Z_2) - Z_m^2} \quad (73)$$

Had no transformer been used, the current through the source and load would have been:

$$I_L = \frac{E}{Z_s + Z_L} \quad (74)$$

By comparing or establishing the ratio of these two currents, it

can be determined how much improvement was obtained from the placing of the transformer. Without establishing quantities, we can investigate other features of the effect of connecting the transformer.

For example, if we assume a transformer with no losses and a very high impedance winding, then:

$$Z_m = \sqrt{Z_1 Z_2} \quad (75)$$

Further, let 'a' equal the ratio of  $\frac{Z_2}{Z_1}$ . Then:

$$Z_2 = Z_1 a \quad (76)$$

$$Z_m = \sqrt{Z_1 Z_2} = Z_1 \sqrt{a} \quad (77)$$

Since the self impedances of the windings are large compared to the terminal (source and load) impedances, then:

$$Z_1 \text{ or } Z_2 > Z_s \text{ or } Z_L \quad (78)$$

Using these facts in Formula 73, we obtain:

$$\begin{aligned} I_L &= \frac{-E Z_1 \sqrt{a}}{Z_1 (a Z_s + Z_L) + Z_s Z_L} \approx \frac{-E \sqrt{a}}{a Z_s + Z_L} \\ &= \frac{-E \sqrt{a}}{a (R_s + jX_s) + (R_L + jX_L)} \end{aligned} \quad (79)$$

where the symbol  $\approx$  means approximately equal to.

Because the polar form equals the rectangular form:

$$R_s + jX_s = \sqrt{R_s^2 + X_s^2} \text{ and } R_L + jX_L = \sqrt{R_L^2 + X_L^2} \quad (80)$$

These then may be substituted in Formula 79:

$$I_L = \frac{E \sqrt{a}}{\sqrt{(R_L + a R_s)^2 + (X_L + a X_s)^2}} \quad (81)$$

By applying simple calculus to this Formula, with respect to a, the scalar value of  $I_L$  can be shown to be maximum when:

$$a = \frac{Z_2}{Z_1} = \sqrt{\frac{R_L^2 + X_L^2}{R_s^2 + X_s^2}} = \frac{Z_L}{Z_s} \quad (82)$$

This equation shows that in a lossless transformer, the maximum transfer of energy occurs when the ratio of the self impedances of the transformer windings equals the ratio of the scalar values of the load and of source impedances.

If next we add to the scalar values of  $Z_L$  and  $Z_s$  the angle, so as

to make these polar forms, and substituting the values of Formula 82 in Formula 79, we obtain:

$$I_L (\max) = \frac{E\sqrt{a}}{aZ_s + Z_L} = \frac{E}{Z_s \sqrt{\frac{Z_L}{Z_s}} + Z_L \sqrt{\frac{Z_s}{Z_L}}} \quad (83)$$

Formula 83 is in scalar form. By adding the angles to convert it to polar form, we obtain:  $Z_L = Z_L / x$  and  $Z_s = /y$

$$I_L (\max) = \frac{E}{\left( Z_L \cos x \sqrt{\frac{Z_s}{Z_L}} + Z_s \cos y \sqrt{\frac{Z_L}{Z_s}} \right)} + j \frac{E}{\left( Z_L \sin x \sqrt{\frac{Z_s}{Z_L}} + Z_s \sin y \sqrt{\frac{Z_L}{Z_s}} \right)} = \frac{E}{2\sqrt{Z_L Z_s} \cos \frac{(x - y)}{2}} \quad (84)$$

Formula 84 shows the maximum value of the current through the load when a perfect transformer connects the source to the load. However, the current could have been further increased if there had not been current-limiting reactance in series with the source and with the load. Had the reactance been corrected by placing complements of their reactance (reducing their reactance to zero) in series with the source and load before inserting the transformer, then the current would have been:

$$I_L (\text{phase corrected}) = \frac{E}{2\sqrt{R_s R_L}} \quad (85)$$

and the power in the load would have been:

$$\text{watts (maximum with phase correction)} = \frac{E^2}{4R_s} \quad (86)$$

Formula 86 states that the greatest amount of power that can be taken from a source by a load occurs when the reactances are reduced to zero and a perfect (no loss) transformer with self impedances matching the source and load is connected between them. Reactances must be reduced to zero because they limit the amount of power flowing in the circuit. The reactances do not use power (only resistances or equivalent resistances can use power) but the reactances do limit the current flow and therefore the amount of power that can be transferred.

To study the amount of power lost by not placing a perfect transformer between a source impedance and a load impedance without first correcting for phase angle in these impedances, a ratio of the maximum current with and without the transformer is ob-

tained from Formulas 84 and 74:

$$\begin{aligned} \text{Loss} &= \frac{I_L \text{ without transformer}}{I_L \text{ with transformer}} = \frac{\frac{E}{Z_s + Z_L}}{\frac{E}{2\sqrt{Z_L Z_s} \cos \frac{x-y}{2}}} \\ &= \frac{2\sqrt{Z_L Z_s}}{Z_s + Z_L} \cos \frac{x-y}{2} \end{aligned} \quad (87)$$

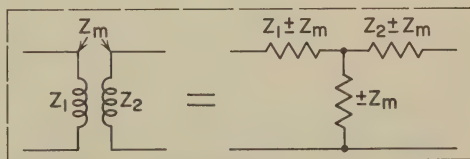
The first factor of this formula  $2\sqrt{Z_L Z_s}/Z_s + Z_L$  shows the loss due to unmatched magnitudes or scalar quantities. This is sometimes called the *reflection loss factor*. The second factor,  $\cos x-y/2$ , shows the loss due to the difference in the angles of the source and load impedances (*the phase-difference loss*). Therefore, the total gain obtained from inserting a perfect transformer between the two impedances consists of the gain that overcomes the reflection loss and the gain that overcomes the phase-difference loss.

Another interesting feature of transformers is their reduction to an equivalent network that is easier to handle than the complex transformer formulas. This is done by using Thevenin's method. The method might be stated loosely: if the impedance of the circuit looking *through* the transformer can be determined, it would make little difference to the source whether the transformer and load were connected to it or whether an impedance of similar value were connected in its place. Similarly, if the impedance and the voltage source, looking from the load impedance through the transformer were to be replaced by similar impedance and voltage values, the load impedance could not be affected by the replacement.

This principle has been worked out and the equivalent circuit in the form of a T network is shown in Fig. 21. The impedance looking either way through the network of the transformer is the same.

In order to gain familiarity with computations of this type, the solution of a relevant problem is now presented. Assume the source

**Fig. 21.** Equivalent network of a transformer.



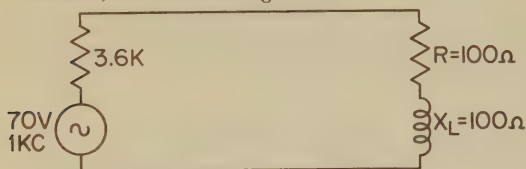
voltage and load are given, and connected to the transformer. Now, determine (a) the proper turns ratio and (b) the gain or loss due



to the insertion of the transformer between the source and load.

### Illustrative

**Problem 15:** Our assumed values are: a source of 70 volts in series with an impedance of 3600 ohms, the voltage having a frequency of 1000 cycles. The load consists of a 100-ohms resistive load in series with a coil having 100 ohms reactance, as shown in Fig. 22.



**Fig. 22.** Simple circuit for inserting impedance-matching transformers.

**Solution:** Let us first determine the power that is delivered to the load without the transformer, since this will serve as the base for a comparison of the benefits of connecting the two impedances by means of a transformer.

$$W = I^2 R \quad (88)$$

where  $E$  is 70 volts:

$$I = \frac{E}{Z} = \frac{70}{3600 + 100 + j100} = \frac{70}{3700 + j100} \quad (89)$$

from Formulas 12 and 14:

$$\begin{aligned} I &= \frac{70}{3700 + j100} = \frac{70}{\sqrt{3700^2 + 100^2}} \frac{1}{(\text{and the angle whose tangent is } .0270)} \\ &\approx \frac{70}{3700 \angle 1.5} = .0189 \angle 1.5 \end{aligned} \quad (90)$$

Substituting this value in Formula 88.

$$W = I^2 R = .0189^2 \times 100 = .00035 \times 100 = .035 \text{ watt} \quad (91)$$

The computation indicates that without a transformer the power delivered to the load is .035 watt. It will now be our problem to determine, (a) the transformer that should have been used, (b) its turns ratio and (c) the delivered power.

From Formula 60, the self impedances, or the impedances of the circuit looking from one side of the transformer through the transformer to the impedance on the other side, should equal the impedance facing that winding. Referring back to Fig. 22, we find the impedance looking toward the load  $100 + j100$ , equivalent to  $141/45$ , should equal 141 ohms times the turns ratio of the transformer that would step this impedance up to 3700 ohms.

This may be expressed, as in Formula 63:

$$\frac{T_1}{T_2} = -\sqrt{\frac{Z_1}{Z_2}} = -\sqrt{\frac{3600}{141}} = -\sqrt{25.1} = -5.1 \quad (92)$$

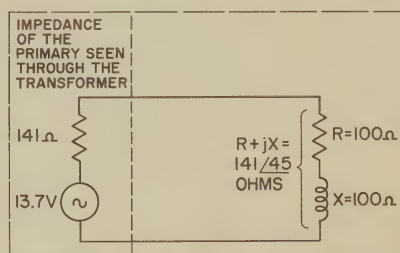
The turns ratio of the transformer should be 5.1 to 1, the minus sign indicating that the phase difference in the voltages on the two sides of the transformer is  $180^\circ$ . The power delivered through this 5.1-turns transformer, which is assumed perfect (lossless), may now be computed from Formula 88 where  $I$  represents the current through the actual load and  $R$  the scalar value of the load 141 ohms. In calculating this value, it is first necessary to develop the voltage at the secondary or load side of the transformer.

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} \quad (\text{from Formula 63}) \quad (93)$$

$$E_2 = \frac{E_1 T_2}{T_1} = \frac{70}{5.1} = 13.7 \text{ volts} \quad (94)$$

The computation indicates the secondary impedance of  $141/45$  ohms has a voltage of 13.7 volts acting across it, as shown in Fig. 23.

**Fig. 23.** Equivalent circuit of the secondary, or load side, of the circuit.



The current flowing in the circuit of Fig. 23 may be computed:

$$I = \frac{E}{Z} = \frac{13.7}{241 + j100} = \frac{13.7}{262/22} = .052/22 \text{ amperes} \quad (95)$$

By substituting the results of Formula 95 in Formula 88 the power in the load now is:

$$w = I^2 R = .052^2 \times 100 = .0027 \times 100 = 0.27 \text{ watt} \quad (96)$$

This is a considerable increase in power over the case without the transformer, as computed in Formula 91. The ratio of these two powers is:

$$\frac{w \text{ after}}{w \text{ before}} = \frac{0.27}{.035} = 7.7 \quad (97)$$

The gain now can be expressed in db in the following manner:

$$\begin{aligned} \text{No. of db} &= 10 \log \frac{w \text{ after}}{w \text{ before}} = 10 \log 7.7 = 10 \times 0.89 \\ &= 8.9 \text{ db} \end{aligned} \quad (98)$$

The insertion of a transformer in the circuit has increased the output by 8.9 db. Db is nearly always a ratio of power measured in logarithmic units in electrical problems. Here it measures a ratio of the power referred to the condition before the insertion of the transformer.

If it is possible to tune out the load reactance, the numbers in Fig. 23 change with a revised 6:1 turns ratio to 70/6 volts for the generator, 100 ohms for the generator impedance, and 100 ohms for the resistive load. In this case the power in the load is 0.34 watt, which is the maximum that can be obtained.

The previous problem gives transformer considerations on the basis of a perfect or lossless transformer. It is reiterated that this is permissible; most modern transformers have little loss.

**Reflected Impedance.** The computation of impedance looking through the transformer described above, and diagrammed in Fig. 23, might have been solved by a different set of formulas. The amount of impedance that appears at the first winding as the result of the second winding being closed through some circuit is called reflected impedance. The formula is:

$$Z_r = \frac{(2\pi f m)^2}{Z_2} \quad (99)$$

Where  $Z_r$  is reflected impedance,  $f$  is the frequency at which computations are made,  $m$  is the inductance of the mutual effects between the windings, and  $Z_2$  is the second winding impedance through which current flows. The  $Z_2$  includes the impedance of the winding and the circuit through which the winding current path is completed.

The total impedance at the first winding includes the impedance of that winding and the reflected impedance:

$$Z_t = Z_1 + Z_r \quad (100)$$

Where  $Z_t$  is the total impedance from the first winding side,  $Z_1$  is the impedance of the first winding, and  $Z_r$  is the impedance of Formula 99. These impedance values are polar or rectangular, not scalar; they include both resistive and reactive components. Normally, the impedances of  $Z_1$  and  $Z_r$  are inductive, that is  $R + jX$ . As a result, when the impedance  $Z_r$  is brought through the transformer the phase becomes inverted. Therefore:

$$Z_r = R_r - jX_r = \frac{(2\pi f m)^2 R_2}{R_2^2 + X_2^2} - j \frac{(2\pi f m)^2 X_2}{R_2^2 + X_2^2} \quad (101)$$

A case to describe this formula is given in Fig. 24:

1. Impedance of winding 2:

$$\begin{aligned} Z_2 &= R + jX_L - jX_c = 20 + 30 \\ &+ j\left(6.3 \times 1000 \times 0.1 - \frac{10^6}{6.3 \times 1000 \times 1}\right) \\ &= 50 + j(630 - 160) = 50 + j470 = 472/\underline{83.9} \quad (102) \end{aligned}$$

2. Impedance of winding 2 reflected into winding 1:

$$Z_r = \frac{(2\pi f m)^2}{Z_2} = \frac{(6.3 \times 1000 \times 0.08)^2}{472/\underline{83.9}} = \frac{250000}{472/\underline{83.9}} = 532/\underline{83.9} \quad (103)$$

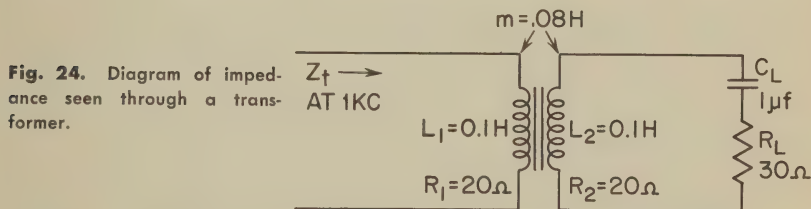
3. The impedance of the load as seen through the transformer:

$$\begin{aligned} Z_t &= Z_1 + Z_r = 20 + j630 + 532/\underline{83.9} = 20 + j630 \\ &+ 56 - j526 = 76 + j104 = 129/\underline{53.8} \quad (104) \end{aligned}$$

It is interesting to note that winding 1 which had only 20-d-c ohms now has an impedance of 129 ohms, even more than the sum of the d-c components of windings 1 and 2 and the load resistance.

In determining the impedance of a load, therefore, it is necessary to calculate the effect of the transformer on the entire circuit, making a full computation of the impedances all the way through the circuit.

Transformers are convenient for changing the impedance of a load to match a source. In the example of Fig. 24, however, not fully computing the effect of the transformer might give the impression that placing a transformer between two equal impedances



(because it is a 1:1 turns ratio) would not affect the matching. In this case the matching is definitely affected. If the source is similar to the load circuit, (a one  $\mu f$  capacitor in series with 30 ohms), the source faces not 30 minus  $j160$  ohms, but the total impedance of 129/53.8. This would mean an impedance mismatch. The source impedance would be:

$$30 - j160 = 163/\underline{79.4} \quad (105)$$

which together with the impedance  $Z_1$ , faces the load as seen through the transformer of

$$Z_r = 532/\sqrt{83.9} \quad (106)$$

and the voltage  $E_1$  (see Fig. 17) for a source voltage of 100 volts would be:

$$E_1 = \frac{100 \times 129/\sqrt{53.8}}{(30 - j160) + (76 + j104)} = \frac{12,900/\sqrt{53.8}}{106 - j56} = \frac{12,900/\sqrt{53.8}}{120/\sqrt{27.9}} = 107 \text{ volts at } 81.7^\circ \quad (107)$$

In order to see how much power is transferred to the load  $Z_L$ , one must now find the secondary voltage  $E_2$  across  $Z_L$ . From Formulas 53, 54 and

$$E_2 = Z_L I_2 \quad (108)$$

one can show that the relation between  $E_1$  and  $E_2$  is given by

$$E_2 = - \frac{Z_L Z_m}{Z_1 (Z_2 + Z_L) - Z_m^2} E_1 \quad (109)$$

Substitution of previously calculated quantities into the right-hand side of Formula 109 shows that

$$E_2 = 1.34/\sqrt{53} E_1 = (1.34/\sqrt{53}) (107/\sqrt{81.7}) = 143/\sqrt{134.7} \quad (110)$$

The current  $I_2$  through  $Z_L$  is now found to be

$$I_2 = \frac{E_2}{Z_L} = \frac{143/\sqrt{134.7}}{163/\sqrt{79.4}} = -0.877/\sqrt{34.1} \quad (111)$$

The power dissipated in the load resistor  $R_L$  is given by

$$W_L = R_L I_2^2 = 30 (0.877)^2 = 23 \text{ watts} \quad (112)$$

If no transformer is used, then  $Z_s$  and  $Z_L$  are the same so that the current  $I$  is given by

$$I = \frac{E_s}{2Z_L} = \frac{100}{2(30 - j160)} = \frac{50}{163/\sqrt{79.4}} = 0.307/\sqrt{79.4} \quad (113)$$

A calculation of the power dissipated in  $R_L$  shows that

$$W_L = R_L I^2 = 30 (0.307)^2 = 2.8 \text{ watts} \quad (114)$$

This amount is considerably smaller than that in Formula 112. The transformer helps in a partial matching of impedances. The maximum power available from the source occurs when the reactance of the load cancels out the reactance of the source. For



this case one should use

$$Z_L = 30 + j160 \quad (115)$$

The current is now

$$I = \frac{E_s}{(30 - j160) + (30 + j160)} = \frac{100}{60} = \frac{5}{3} \text{ amperes}$$

and the power dissipated in  $R_L$  is now

$$W_L = R_L I^2 = 30 \left( \frac{5}{3} \right)^2 = 83.3 \text{ watts}$$

It is apparent that the power dissipated in the load in the presence of the transformer is much greater than in its absence but is at the same time quite a bit smaller than the maximum available power.

## 9. Cathode-Follower Circuitry

The use of transformers for impedance matching has been shown to be valuable for the gains they effect in coupling load circuits to source circuits. Like several other devices used for this purpose, however, transformers usually load the circuit in which the source voltage is found.

Loading in this sense means that the voltage available at the source is affected by the impedance of the circuit connected to it; usually, the voltage available at the load is reduced. For example, if a source of 100 volts and 100 ohms is connected to a load circuit of 100 ohms, the voltage across the load would be only 50 volts. That is, the load "loaded" down the source voltage from 100 volts by requiring enough current to cause a drop of 50 volts across the source resistance, making only 50 volts available to the load circuit. If the load circuit had been a megohm in value, the voltage across it would have been very nearly 100 volts, because a megohm across a hundred ohms is not much of a load (does not permit much current flow so little is lost across the source resistance).

Now it so happens that an electronic device is readily available that has a high input impedance, and hence does not appreciably load the circuit preceding it. At the same time this device, called a cathode follower, has a low output impedance. This and other advantages, soon to become apparent, make the cathode follower a popular circuit.

Figure 25 illustrates a simple cathode follower. Note that the plate is tied directly to B+ while the entire output signal appears

across the cathode resistor, ( $R_k$ ). The plate current (initiated by the input signal) takes the path as indicated in the illustration. As can be seen, the effective input voltage between grid and cathode will be the difference between the total input signal ( $E_{in}$ ) and the signal dropped across the cathode resistance ( $E_k$ ).

Since the output signal occurs across  $R_k$ , and  $R_k$  is in series with the grid circuit, the effective input signal voltage appearing between grid and cathode is the vector sum of the input + output signal. This feeding of the output signal back to the input is known as feedback.

The voltage across  $R_k$  is in phase with  $E_{in}$ ; that is, there is no phase reversal (as occurs in most amplifiers). However, along with this in-phase condition, when the grid starts to rise in the positive

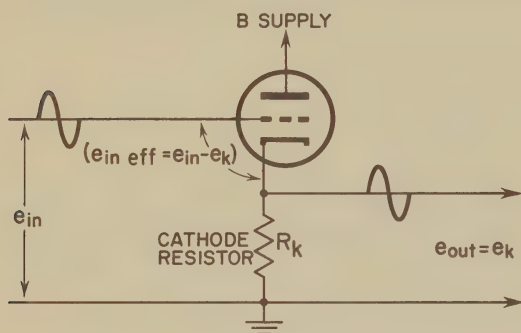


Fig. 25. Circuit of a simple triode cathode follower.

direction (due to the action of  $E_{in}$ ),  $E_k$  also rises in the positive direction. The action has the effect of *reducing* the effective signal voltage between grid and cathode. This type of feedback, then, is called negative feedback. Negative feedback in this and in other systems serves to lessen the gain (or increase the loss) of the system, as well as to lessen the amount of distortion contributed to the signal by the device.

The output impedance is predominantly controlled by the value of  $R_k$ .

To review, the circuit of Fig. 25 may be described as follows: the grid circuit must be closed through some d-c path to provide continuity of the biasing arrangement. Sometimes this circuit may also include arrangements for adding biases, as for example if a battery were made part of the return to grid path. The voltage gain of the circuit is always less than one because the grid circuit receives not only the input signal but also a  $180^\circ$  out-of-phase voltage from the cathode resistor. This usually has a value that makes the maximum gain of the tube circuit about 0.9, resulting in an output voltage which is always less than the input voltage. Because

these two voltages are out of phase the grid will not draw current until the input voltage exceeds the bias. Out of phaseness also causes the input impedance to be quite high and the output impedance to be low. Because the input is high and output impedance is low, the circuit is often thought of as an impedance-matching device.

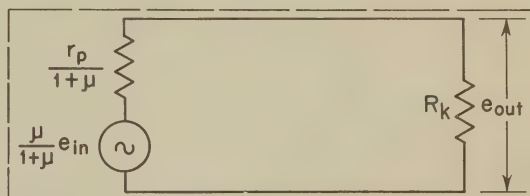
At low frequencies, where the shunt capacitances of the tube and circuit elements are not important, we can express the gain as:

$$G \text{ (no feedback)} = \frac{\mu R_k}{R_k + r_p} \quad (116)$$

where  $G$  is gain,  $\mu$  is mu,  $R_k$  is the ohmic value of the cathode resistor, and  $r_p$  is the internal plate-cathode resistance. No feedback here means the application of input voltage ( $E_{in}$ ) directly across grid to cathode.

The portion of the output voltage that is fed back to the input is called the transfer coefficient or feedback factor. In this case

**Fig. 26.** Equivalent circuit of a cathode follower.



all of the output is fed back to the input. It is also out of phase with the input and may therefore be designated as  $b$  having a value of  $-1$ . The gain with feedback modifies Formula 116 and it should read:

$$G \text{ (with feedback)} = \frac{G}{1 - bG} = \frac{G}{1 + G} \quad (117)$$

Formula 117 may be combined with Formula 116 as follows:

$$G \text{ (with feedback)} = \frac{\mu R_k}{(\mu + 1) R_k + r_p} \quad (118)$$

As  $g_m$  is  $\frac{\mu}{r_p}$ , then

$$G \text{ (with feedback)} = \frac{R_k}{\frac{(\mu + 1)}{\mu} R_k + \frac{1}{g_m}} \quad (119)$$

If the  $\mu$  of the tube is as great as with a high- $\mu$  triode, Formula 119 can be reduced to an approximate form:

$$G \text{ (with feedback and high-}\mu \text{ tube)} = \frac{g_m R_k}{1 + g_m R_k} \quad (120)$$

### Illustrative

Problem 16: The 6SQ7 is a high- $\mu$  triode. It has a transconductance of approximately 1000  $\mu$ mhos.

Compute the gain with  $R_k$  values of

$$(a) 100 \, \Omega \quad (b) 1000 \, \Omega \quad (c) 10,000 \, \Omega$$

Solution:

$$(a) \quad G = \frac{g_m R_k}{1 + g_m R_k} = \frac{.001 \times 100}{1 + .001 \times 100} = .09$$

$$(b) \quad G = \frac{g_m R_k}{1 + g_m R_k} = \frac{.001 \times 1000}{1 + .001 \times 1000} = .5$$

$$(c) \quad G = \frac{g_m R_k}{1 + g_m R_k} = \frac{.001 \times 10,000}{1 + .001 \times 10,000} = .91$$

Assuming an input voltage that causes neither grid current flow nor plate current cut off, Formula 120 tells us that the gain of the tube is not increased by raising the value of the cathode resistor above approximately twice the plate resistance. With tubes having  $\mu$  values of more than 10, the increase of the cathode resistor above twice the plate resistance does not increase the output more than 1/20 more.

For circuit calculations the cathode follower may be represented by the equivalent circuit of Fig. 25B in which a source impedance of

$$\frac{r_p}{1 + \mu} \quad (121A)$$

in series with a generator voltage of

$$\frac{\mu}{1 + \mu} e_m \quad (121B)$$

facing the cathode resistor  $R_k$ .

If  $\mu$  is great compared to one, then Formula 121A shows that the source impedance is approximately

$$\frac{r_p}{\mu} = \frac{1}{g_m} \quad (121C)$$

The output impedance looking back from the point marked "out" in Fig. 25 may be found by setting  $e_{in}$  equal to zero and adding the two resistors in parallel. The results are:

$$R \text{ (output)} = \frac{R_k r_p}{(\mu + 1) R_k + r_p} \quad (122)$$

Assuming  $r_p$  is very large compared to  $R_k$  then:

$$R \text{ (output)} = \frac{R_k}{1 + g_m R_k} \quad (123)$$

### Illustrative

**Problem 17:** The following problem shows how Formula 123 may be used: Using a 6SG7 tube with cathode resistances of

- (a)  $50 \, \Omega$       (b)  $100 \, \Omega$       (c)  $1000 \, \Omega$

the  $R$  output is:

**Solution:**

$$(a) \quad R \text{ (output)} = \frac{R_k}{1 + g_m R_k} = \frac{50}{1 + .001 \times 50} = 47.6 \, \Omega$$

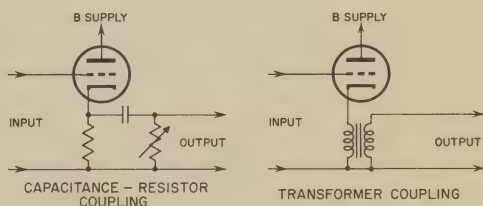
$$(b) \quad R \text{ (output)} = \frac{R_k}{1 + g_m R_k} = \frac{100}{1 + .001 \times 100} = 90.9 \, \Omega$$

$$(c) \quad R \text{ (output)} = \frac{R_k}{1 + g_m R_k} = \frac{1000}{1 + .001 \times 1000} = 500 \, \Omega$$

Cathode followers are sometimes arranged with the cathode resistor followed by a coupling capacitor and another resistor, or a transformer, used to couple the output to the next circuit.

The cathode follower is such an important arrangement for changing impedance relationships between two circuits that both

**Fig. 27.** Two types of cathode-follower output circuits.



the input and output circuits have been made with a large variety of connections. These provide special adjustments of the grid bias, the input and output impedance, and the isolation of the circuits feeding and being fed by the cathode follower.

Figure 28 shows a circuit where only a portion of the cathode resistance is used to produce the grid bias. In this circuit the resistors  $R_a$  and  $R_b$  are adjusted to give the best operating bias for the circuit. If the current flowing through  $R_g$  is ignored because it is extremely small compared to the current through  $R_a$  and  $R_b$ , the bias voltage is given by:

$$e_k = R_a I_p \quad (124)$$

where  $R_a$  is in ohms and  $I_p$  is the plate current in amperes.



The input impedance can be calculated by finding the input capacitance and the input resistance. The input capacitance is given by the equation:

$$C_i = C_{gp} + C_{gk} \frac{(e_i - e_k)}{e_i} \quad (125)$$

in which  $C_i$  is the input capacitance,  $C_{gp}$  is the interelectrode capacitance between grid and plate, and the voltages are those shown

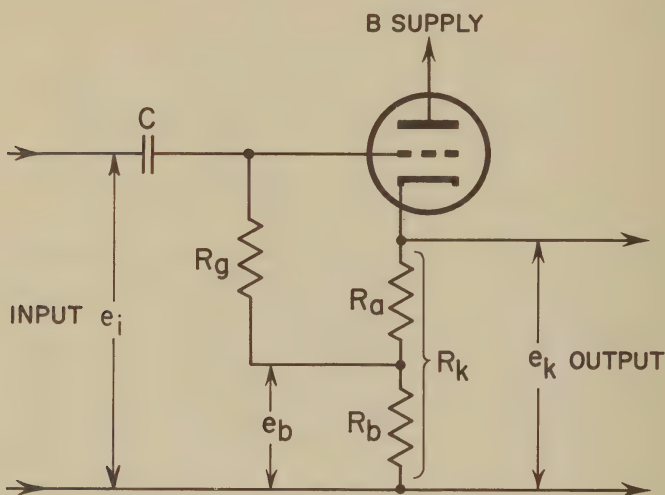


Fig. 28. Schematic of a cathode-follower circuit with an adjusted base.

in Fig. 28. For most practical purposes,  $C_i$  may be taken as equal to  $C_{gp}$  since the second factor in the equation is very small.

The resistive portion of the input impedance, in parallel with the input capacitance, is  $R_g (e_i / e_i - e_b)$ . Equation 119 may be rewritten in the form given in Equation 126, taking into account the assumption that the current in  $R_g$  is small enough to be ignored and that the cathode resistor comprises  $R_a + R_b$ . Thus,

$$\text{Gain} = \frac{e_k}{e_i} = \frac{1}{1 + \frac{1}{\mu} + \frac{1}{g_m (R_a + R_b)}} \quad (126)$$

Also, from simple voltage divider considerations:

$$\frac{e_b}{e_k} = \frac{R_b}{R_a + R_b} \quad (126A)$$

Combining Equations 126 and 126A, we obtain:

$$\frac{e_i}{e_i - e_b} = \frac{1}{1 - \frac{e_b}{e_i}} = \frac{1}{1 - \frac{R_b}{(R_a + R_b) \left(1 + \frac{1}{\mu} + \frac{1}{g_m (R_a + R_b)}\right)}} \quad (126B)$$

In practical circuits,  $\mu$  is generally much greater than unity; the same applies to the factor  $g_m (R_a + R_b)$ . Under these conditions,  $e_i/e_i - e_b$  may be taken as equal to  $R_a + R_b/R_a$  and the approximate input resistance then becomes:

$$R_i = R_g \left( R_a + \frac{R_b}{R_a} \right) \quad (126C)$$

#### Illustrative

**Problem 18:** Find the input resistance to a triode cathode follower stage in which the grid resistor is 0.5 megohms,  $R_a$  is 2000 ohms and  $R_b$  is 1000 ohms.

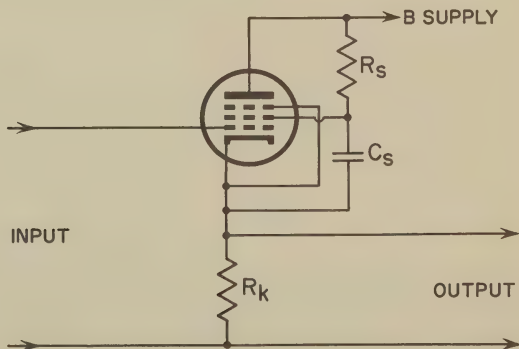
**Solution:** Substituting in equation 126C,

$$R_i = 500,000 (3000/2000) = 750,000 \text{ ohms}$$

The total input impedance  $Z_i$  can be calculated by combining vectorially this value of input resistance with the capacitive reactance of  $C_{gp}$  for the particular tube and at the frequency in which the user is interested.

Pentodes may also be used as cathode followers. An example of one is given in Fig. 29. Here the resistor or choke  $R_s$  is used to drop the screen voltage to the desired value. The capacitor  $C_s$  is returned to the cathode circuit not only to stabilize the screen

**Fig. 29.** Schematic of a cathode-follower circuit using a pentode tube.



voltage but also to keep the variations that occur from affecting the cathode resistor  $R_k$ . In this way, pentode operation is maintained.

The voltage gain for this circuit is:

$$\text{Gain} = \frac{g_m R_k}{1 + g_m R_k} \quad (127)$$

**Illustrative**

**Problem 19:** Let us compute the voltage gain of a 6SJ7 tube with a cathode resistor of 1000 ohms and a transconductance of 1600 micromhos.

**Solution:**

$$G = \frac{1.6 \times 10^{-3} \times 10^3}{1 + 1.6 \times 10^{-3} \times 10^3} = \frac{1.6}{2.6} = 0.62$$

Many cathode follower arrangements have been used in practical circuitry. Essentially, it is an impedance changer which, without appreciably loading the source circuit, controls the impedance presented to the next circuit.

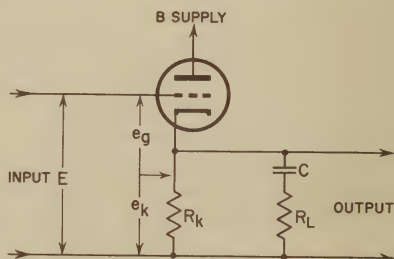
## 10. Cathode Follower in Video Circuits

Often in commercial video installations it is necessary to connect the output of a video frequency power amplifier to a low-impedance coaxial cable. Amplifiers for this purpose present special problems since the load impedance is small (in the order of 50 to 150  $\Omega$ ). The wide band of frequencies involved makes it impossible to use transformers for impedance matching.

The circuit arrangement for a cathode-follower type of video amplifier is shown in Fig. 30.

In this arrangement the d-c plate current is kept from passing through the load resistor by the placement of a coupling capacitor

**Fig. 30.** Schematic of a cathode-follower circuit using video frequencies.



in series with the load. The capacitor has to be large in capacity to avoid limiting the amount of current in the load. Assuming the plate current varies from zero to twice its normal value during signal transmission, the power that can be delivered from a Class A cathode follower circuit to a load resistor is:

$$\text{No. of watts} = \frac{I_p^2 R_L}{2} \quad (128)$$

Actually the power must be smaller than this if distortion is to be kept low. If a great amount of power is needed it can be seen from Formula 128 that the plate current  $I_p$  must be large. This may be accomplished within limits by placing several tubes in parallel.

If dc can be allowed to pass through the line, the output may be delivered directly to a cable, and several advantages accrue. One side of the cable may be grounded. Amplitude and phase characteristics are better because of the negative feedback action. A higher input impedance can be obtained because of the feedback. The load impedance can be matched to the tube by adding either a resistor in shunt of the cable input or in series with one side.

The voltage that may be developed across the load is:

$$E = \frac{g_m Z_L}{1 + g_m Z_L} e_g$$

(if  $Z_L$  includes the cable load and the cathode resistor) where  $E$  is the load voltage,  $g_m$  the transconductance of the tube,  $Z_L$  the actual impedance of the load, and  $e_g$  the grid voltage noted in Fig. 30.

#### Illustrative

**Problem 20:** Two video amplifier stages are to be coupled with a 50  $\Omega$  transmission line. Design the matching output circuit.

**Solution:** In order to match such a low impedance line we would make use of the cathode follower. A pentode tube of type 6AG7 would suit this purpose. The tube characteristics are: transconductance ( $g_m$ ) = 11000  $\mu$ mhos, plate resistance ( $r_p$ ) = 130,000  $\Omega$ , amplification factor ( $\mu$ ) =  $g_m \times r_p = 143$ . To find the value of the cathode resistor ( $R_k$ ) we transpose Formula 122 and solve for  $R_k$ :

$$R_k = \frac{Z_o r_p}{r_p - Z_o (\mu + 1)}$$

$Z_o$  is the impedance of the device which is to be matched. Substituting our values in this equation we get:

$$R_k = \frac{50 \times 130,000}{130,000 - 50(143+1)} = 53 \Omega$$

This tells us that a cathode resistor of 53  $\Omega$  will give us the best matching. A larger cathode resistor would give us more gain, but due to the mismatch some of the frequencies would be attenuated. In transmission of video frequencies it is wise to sacrifice gain for better frequency response. Also, the transmission line should be capacitively coupled to the next stage. If we need a larger tube bias we can add a bypassed resistor in series with  $R_k$ .

## 11. Resistance Networks

Resistance networks are a division of a larger class of circuit arrangements called four-terminal networks. These include any electrical circuitry that has two terminals where the source impedance and power source may be connected, and two other terminals to which a load circuit may be connected.

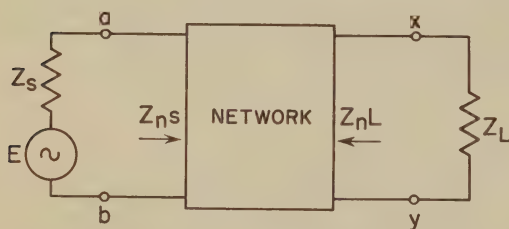
There are a number of ways in which these many circuits may be divided for study. One division is between passive and active networks. Passive networks contain no source of energy.

The properties of a four-terminal network may be expressed, at any one frequency, in terms of three independent factors regardless of how complex the network may be. The commonly used factors are:

1. image impedances and image transfer constant
2. open and short circuit impedances
3. iterative impedances and iterative transfer constant.

*Image Impedances* refer to the impedance looking into the network with the load attached being the same as the impedance of the termination connected to the network at that point. This is diagrammed in Fig. 31.

For example, the impedance looking from terminals a-b toward the network or toward the source, with  $Z_L$  attached to the other end of the network, is the same. Similarly, the impedance looking



**Fig. 31.** Diagram shows arrangement of image impedances.

from the terminals x-y toward the network with the far end closed through  $Z_s$  is the same as the impedance looking toward the load impedance  $Z_L$ .

With the terminations connected to each end of this network,  $Z_{nL}$  equals  $Z_L$ , and  $Z_{ns}$  equals  $Z_s$  which is the condition for image impedances. That is, the network presents the same impedance to the termination as the termination itself has. The image transfer constant  $\Phi$  is described by the formula:

$$\frac{E_L}{E_s} = \frac{Z_{nL}}{Z_{ns}} e^{-\Phi} \quad (128A)$$



Image impedance is an important facility for describing four-terminal networks because it is so extensively used in them. There is no reflection loss at the terminals of a network having image impedance. A simple manner of determining the terminal image impedance values of an image impedance network is given by the formula:

$$Z_n s = Z_n L = \sqrt{Z_{sc} Z_{oc}} \quad (129)$$

( $Z_{sc}$  = short-circuit impedance,  $Z_{oc}$  = open-circuit impedance) which states the impedance that the network images is the square root of the product of its short-circuit and open-circuit impedances. This is also referred to as the surge impedance. The image-transfer constant is the loss of the structure in nepers. A neper is about 8.7 times the size of a decibel. *Iterative impedances* refer to the impedances of a network that match the far-end impedance. For example, referring back to Fig. 31, if the impedance  $Z_n s$  looking through the network equals,  $Z_L$ ,  $Z_n s$  is called the iterative impedance. Similarly if  $Z_n L$  equals  $Z_s$  when looking through the network, then  $Z_n L$  is iterative impedance.

Resistive networks are always passive networks. They always use power in their elements. They do not reduce power losses when used in impedance matching networks as do transformers or cathode followers. They are used in isolating circuits and stabilizing impedances, and to provide attenuation.

**Isolating Circuits.** Where it is desired to keep one circuit from acting on another circuit, isolation filters, circuits, or networks are interposed. The amount of loss caused thereby is usually large, but this is often less important than the need for isolation.

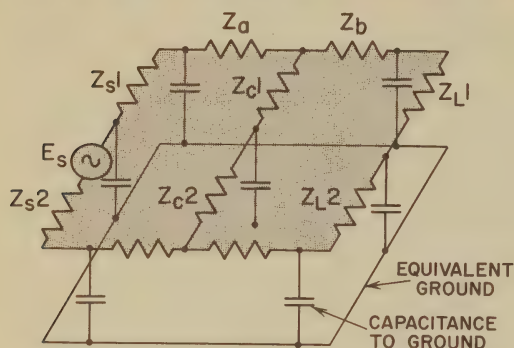
**Stabilizing Impedance.** The output impedance of a circuit (an oscillator, for example) may depend upon reactive components or vacuum tube parameters which, if allowed to determine the output impedance, would seriously affect the operation of the following circuit. To hold the impedance of the circuit rigidly constant, a large attenuation pad or loss pad is placed at the output of the circuit, thus holding the impedance at the nominal impedance of the pad. An approximately 10- to 20-db pad is frequently used for this purpose.

**Providing Loss.** Where circuitry requires that levels be held at certain points along a circuit, variable attenuation or loss networks are used to control the level. The same networks also serve to maintain impedance relationships.

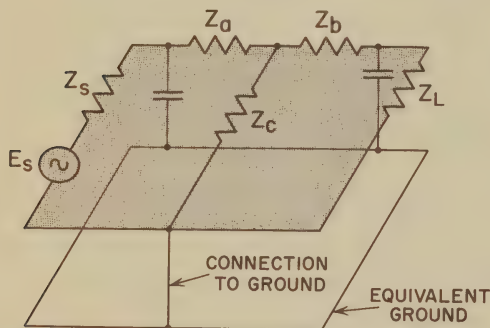
For most purposes there are only a few types of networks or

pads. These are separated into two groups known as balanced and unbalanced networks, as shown in Fig. 32.

In balanced networks the circuit is susceptible to spurious voltages being induced from nearby circuitry, but because the induced voltages tend to be equal on the two sides of the circuit, the net



BALANCED NETWORK HAVING EQUAL ADMITTANCE TO GROUND



UNBALANCED NETWORK HAVING CAPACITANCE TO GROUND ON ONE SIDE AND DIRECT CONNECTION TO GROUND ON THE OTHER SIDE

**Fig. 32.** Balanced and unbalanced networks.

induced voltage is very small. In unbalanced networks, one side of the circuit is "live" and the other "dead" with respect to induced voltages, assuming that nothing is induced in the grounded side. Consequently the induced voltage appears in the output of the circuit.

The term "ground" in this sense means that the surface of the conducting material is used as a reference point. Sometimes the term applies to the actual earth but more often it is the chassis frame or other surface where the physical circuitry is used.

Resistive networks and other networks have many configurations.

The network might be just a potentiometer by itself. It might have one of the common forms called T, H, pi, C, lattice or bridge types. There are even L types of C types. The T, pi, and L types are unbalanced forms, the H, O, and C types are balanced. The lattice or bridge type may be either balanced or unbalanced. Each of these will be discussed separately. The T and H types are shown in Fig. 33.

The T and H pads are different only in the distribution of the resistive values, not in the total resistance, on either side of the

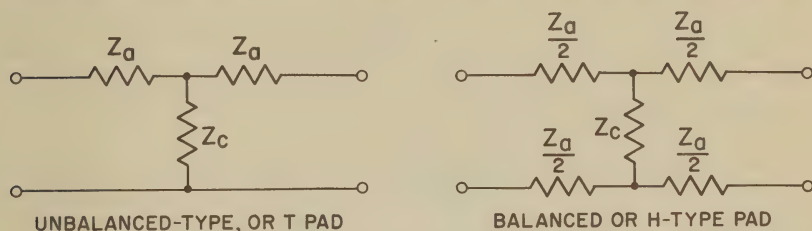


Fig. 33. Representations of T and H pads.

shunt resistor. For this reason the H pad is shown with half  $Z_a$  values.

The effect of inserting a pad in a circuit is expressed in terms of an image impedance, and the amount of loss the network would insert into a properly matched circuit. An example is shown for a T pad in Fig. 34.

In Fig. 34, the values of the resistors may be taken as:

$$\begin{array}{lll} Z_a = 260 \text{ ohms} & Z_s = 500 \text{ ohms} & E = 100 \text{ volts} \\ Z_c = 352 \text{ ohms} & Z_L = 500 \text{ ohms} & \end{array}$$

In the following steps the loss of the circuit in db will be com-

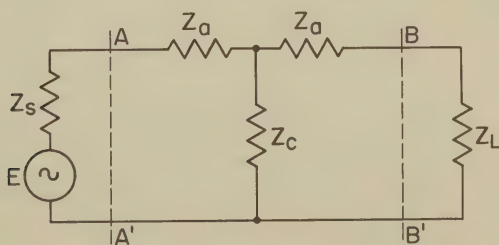


Fig. 34. Arrangement of a T pad.

puted. This will be done in detail to show the computation. First the portion of the voltage across  $Z_c$  that appears at  $Z_L$ :

$$E_L = \frac{Z_L E_c}{Z_L + Z_a} = \frac{500 E_c}{500 + 260} = 0.658 E_c \quad (130)$$

Second, the impedance of the parallel branches consisting of  $Z_c$  in parallel with the resistors  $Z_a$  and  $Z_L$ :

$$Z_p = \frac{Z_c (Z_a + Z_L)}{Z_a + Z_c + Z_L} = \frac{352 \times 760}{260 + 352 + 500} = 240 \text{ ohms} \quad (131)$$

Third, the voltage across  $Z_c$ :

$$E_c = \frac{Z_p E}{Z_p + Z_a + Z_s} = \frac{240 \times 100}{240 + 260 + 500} = 24 \text{ volts} \quad (132)$$

Fourth, the voltage across  $Z_L$ :

$$E_L = \frac{500 \times 24}{760} = 16 \text{ volts} \quad (133)$$

Fifth, the power expended in  $Z_L$ :

$$W = \frac{E_L^2}{Z_L} = \frac{16^2}{500} = 0.5 \text{ watt} \quad (134)$$

Sixth, the voltage across  $Z_L$  if the network had not been used:

$$E_L = \frac{Z_L E}{Z_L + Z_s} = \frac{500 \times 100}{500 + 500} = 50 \text{ volts} \quad (135)$$

Seventh, the power expended in  $Z_L$  without the network:

$$W = \frac{E_L^2}{Z_L} = \frac{50^2}{500} = 5 \text{ watts} \quad (136)$$

Eighth, the ratio of powers expressed in db:

$$\text{No. of db} = 10 \log \frac{5}{0.5} = 10 \log 10 = 10 \times 1 = 10 \quad (137)$$

This solution of Fig. 34 shows that the loss of the circuit is 10 db. The values of the resistors could have been arranged either as a T or an H network as discussed for Fig. 33.

This network fulfills the conditions of both image and iterative types of impedances because the source and load resistors have the same value, and because the impedance looking away from the source at AA<sup>1</sup> or toward the source from BB<sup>1</sup> (Fig. 34) is equal to both  $Z_s$  and  $Z_L$ . In other words,  $Z_a + Z_p$  is the same as  $Z_s$  or  $Z_L$ , and each is 500 ohms. T and H networks are also called a transmission pad.

These computations can be replaced by a table of values set up for the  $Z_a$  and  $Z_c$  impedances of Fig. 34 for a specific image impedance. To convert these for use for other image impedances it is necessary only to multiply each figure by the ratio of the image impedances. For example, this table is for 500 ohms of image impedances; for 1 db the value of  $Z_c$  is 4340 ohms. If the value for 1000-ohm image impedance is needed, it is necessary only to recognize that 1000 ohms is twice as large as 500 ohms, or that they are in the ratio of two to one. Then multiply  $Z_c = 4340$  ohms by 2 to get 8680 ohms, and  $Z_a = 28.7$  ohms by 2 to get 57.4 ohms.

TABLE 3

Resistance Values for a T Network Having an Image Impedance of 500 Ohms

Loss in db	$Z_a$	$Z_c$	Loss in db	$Z_a$	$Z_c$
1	28.7	4340	15	350	182.0
2	57.3	2160	20	409	101.0
3	85.6	1420	25	447	56.3
4	113.0	1050	30	469	32.0
5	140.0	823	35	483	17.3
6	166.0	670	40	490	10.0
7	191.0	559	45	495	5.03
8	215.0	473	50	497	3.16
9	238.0	406	55	498	2.00
10	260.0	350	60	499	1.00

In this table we can observe one very important feature of the T pad, one which would also apply to the H pad. As the loss in the pad increases, the size of the shunt  $Z_c$  becomes smaller. As a result, when the loss of the pad increases, it becomes difficult to construct a pad for any except laboratory use. The pi and O (square pad) have the forms indicated in Fig. 35. They perform the same basic functions as the T and H pad. In addition, the

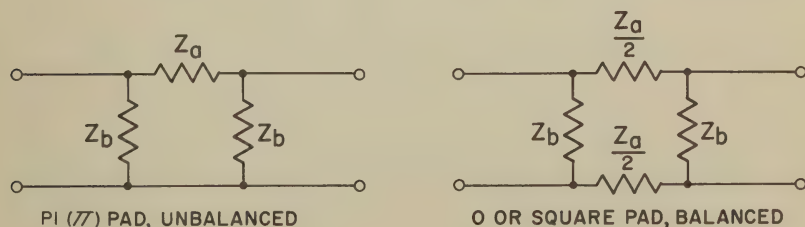


Fig. 35. Representations of pi and O (square) pads.



square pad has a balanced configuration which can more accurately match the characteristics of a balanced transmission line.

TABLE 4

Resistance Values for a Pi Network Having an Image Impedance of 600 Ohms

Loss in db	$Z_a/2$	$Z_b$	Loss in db	$Z_a/2$	$Z_b$
1	34.6	10,400	15	816	860
2	70.0	5215	20	1490	733
3	105.0	3530	25	2610	673
4	143.0	2670	30	4760	639
5	183.0	2140	35	8330	622
6	224.0	1810	40	15,000	612
7	269.0	1570	45	25,900	607
8	317.0	1390	50	45,100	604
9	370.0	1260			
10	427.0	1155			

Again it is evident that the series resistor becomes quite large at high-loss values and the shunt resistor large at low-loss values.

**Lattice or bridge types** of pads are very common. As they are usually drawn they look quite complex. We show an arrangement which makes the appearance simple. An illustration of each type is given in Fig. 36.

These pads use the null effect of a Wheatstone bridge circuit. When perfectly balanced the loss between the input and output

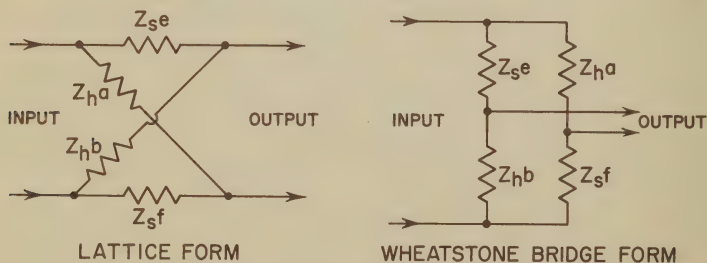


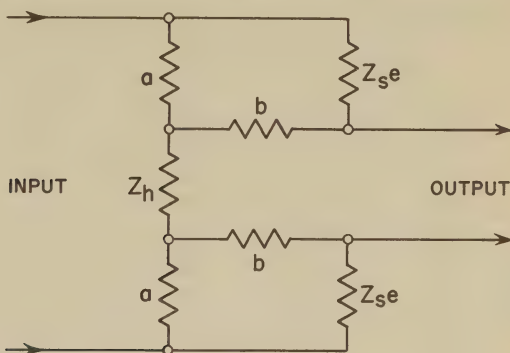
Fig. 36. Representations of lattice- and bridge-forms of pads.

is infinite. As the balance is disturbed the loss becomes smaller. The circuit belongs to the general class previously referred to as balanced networks.

A special circuit variation is used extensively in the communication field not only because it is balanced, but also because it

gives continuity to the circuit even when the variable parts are removed from it. This is shown in Fig. 37.

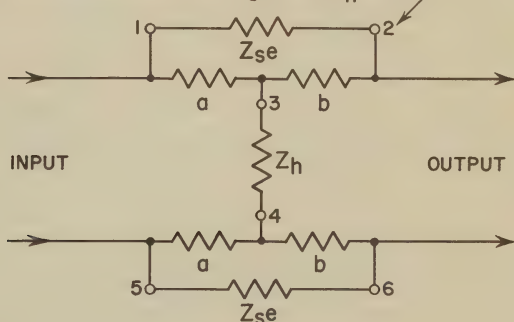
This form of continuity-bridge pad is very convenient because the variable parts  $Z_{se}$  and  $Z_h$  can be assembled in a tube base and used as plug-in units for the basic circuit which has continuity at



#### WHEATSTONE BRIDGE FORM

**Fig. 37.** The balanced continuity-bridge pad.

NUMBERS ON A VACUUM TUBE SOCKET FOR PLUGGING IN VARIOUS  $Z_{se}$  AND  $Z_h$  VALUES



#### CONTINUITY FORM

all times. Thus, with the plug out of the socket, the in and out circuits are closed through the impedances  $a$  and  $b$ . If these two sets of resistances,  $a$  and  $b$ , are each 300 ohms, then the values of the other resistances may be read from Table 5.

By comparing the several forms of T-H, O-pi, and lattice-bridge types, it is obvious that each has a range that is most easily built. Of these several forms the lattice-bridge type has the best field of use because the circuit elements are reasonable in value.

A simple L or split L network appears much like a potentiometer. This is shown in Fig. 38.

TABLE 5

Resistance Values for a Continuity-Bridge Network Having an Image Impedance of 600 Ohms, with the resistors  $a$  and  $b$  of Fig. 37 300 Ohms each

Loss in db	$Z_{se}$	$Z_h$	Loss in db	$Z_{se}$	$Z_h$
1	36.5	4930	11	764	235.0
2	77.8	2320	12	894	201.0
3	124.0	1450	13	1040	173.0
4	175.0	1030	14	1200	150.0
5	233.0	771	15	1390	130.0
6	299.0	602	16	1590	113.0
7	371.0	484	17	1820	98.7
8	454.0	397	18	2080	86.4
9	546.0	330	19	2370	75.8
10	649.0	277	20	2700	66.7

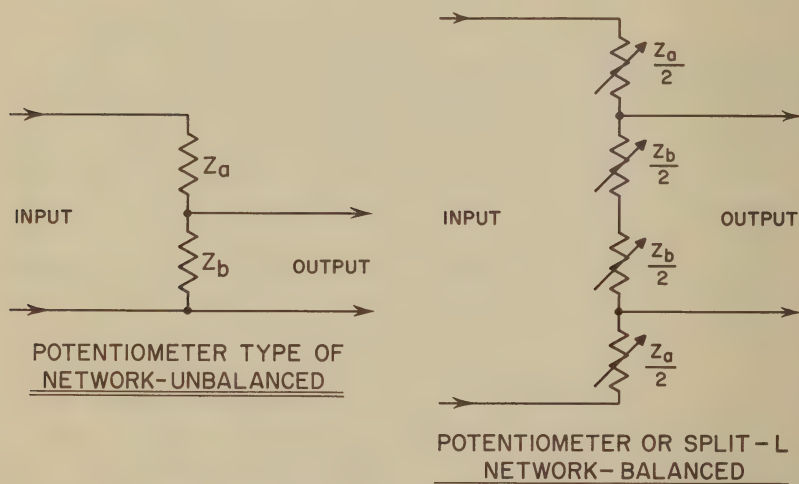
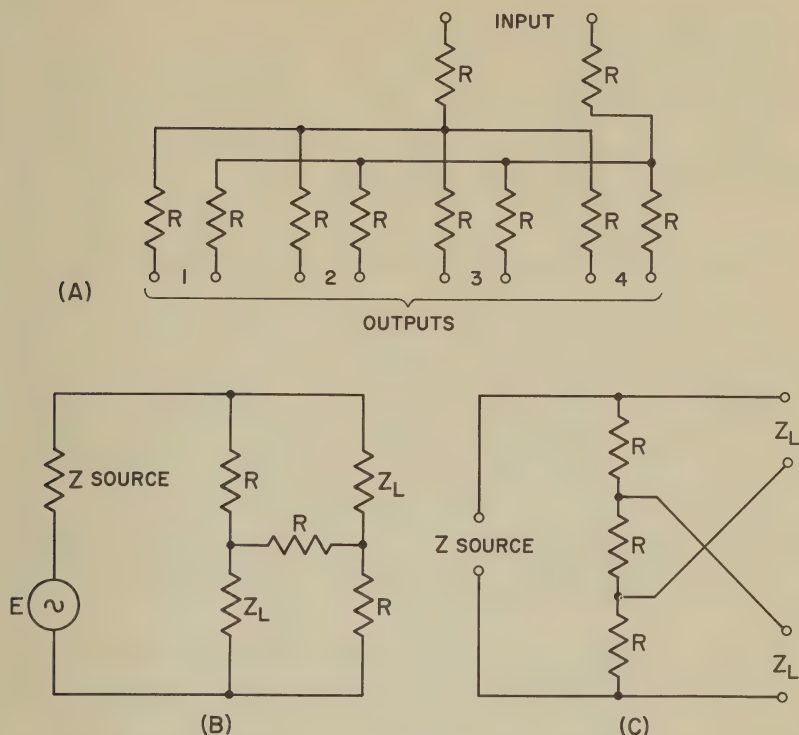


Fig. 38. Schematics of L- or potentiometer-type of network.

The potentiometer type of network has the advantage that it can step down (or up) the impedances on the two sides of the unit.

Resistive networks are also used to provide an impedance match between a number of sets connected to one antenna. These come in many forms. Two types are shown in Fig. 39.

The two types of networks shown in Fig. 39 permit the connecting of a source, such as an antenna, to a number of sets such as radio and TV receivers. It is of course assumed that each set, and



**Fig. 39.** Two examples of resistive-type branching networks used to impedance-match one source with a number of loads.

the antenna, has the same input impedance. Since these are matching impedance networks any unused terminal must be terminated in a matching resistor of similar value, or the impedance of the network would be upset.

A table to compute the required resistance values of the individual elements of these networks is shown below, where  $Z$  is the source impedance,  $N$  is the number of legs or loads used, and  $R$  is the series resistor in each leg. The approximate db loss resulting from the use of the networks is also shown.

The lower circuit is a form of a bridge network and is shown with both the usual bridge type of drawing and the simple sketch of how the three resistors forming it are connected. The signal in this case is attenuated by 6 db, that is it gives isolation to signals from the source to either load, or between either load of 6 db.

If the resistors or the loads are not close to the proper value the isolation feature will be upset.

N number of legs	3	4	5	6	7	8	9	10
Ratio $R/z$ of either load or source	.17	.25	.3	.33	.36	.38	.39	.40
Loss of network	6	10	12	14	16	17	18	19

Part (a) of Fig. 39 may have any number of legs though only four are shown. The lower figure may have only three legs.

## 12. Attenuators Inserted between Unequal Impedances

The formulas and calculations presented in the previous section apply particularly to attenuators inserted in a line between a source impedance and a properly matched load impedance. It is often necessary, however, to choose resistance values for the network which will not only provide the correct value of attenuation loss but which will also serve as an impedance-matching device.

This type of calculation can be extremely tedious if undertaken from the basic equations of the specific network. P. K. McElroy\* has developed a table for the important and recurrent expressions for various values of attenuation which greatly facilitates network design.

To put the tables to use, it is assumed that three quantities are known:  $Z_s$ ,  $Z_L$ , and the desired attenuation in db (symbolized by  $n$ ). Figure 40 gives the most frequently used attenuator networks. The formulas for use with the tables appear on the following pages. As these equations are reviewed, it will be noticed that the quantity " $k$ " appears again and again. It is advisable to indicate the derivation of  $k$  to assist the user in the interpretation of the attenuator equations.

The loss introduced by the insertion of an attenuator is the ratio in db of the power given by the source to the input of the network, to the power delivered to the load by the attenuator. A further qualification of this definition is that the attenuator must be terminated in its image impedance on the load side. This power ratio is conventionally designated as  $k^2$ , so that:

$$k^2 = \frac{\text{Power from source into network}}{\text{Power from network into load}}$$

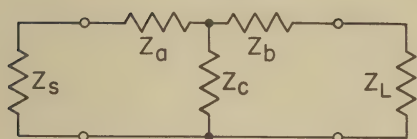
Since the network formulas given in Fig. 40 involve  $k$  in various combinations such as  $k^2 + 1$ ,  $k/k^2 - 1$ ,  $k-1/k+1$ , and so on, the

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\* See pages 64 through 67 for Resistive Attenuating Network Tables.



T - NETWORK

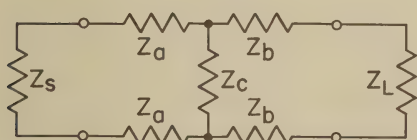


$$Z_a = Z_s \left( \frac{k^2 + 1}{k^2 - 1} \right) - 2\sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

$$Z_b = Z_L \left( \frac{k^2 + 1}{k^2 - 1} \right) - 2\sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

$$Z_c = 2\sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

H - NETWORK



$$Z_a = \frac{Z_s}{2} \left( \frac{k^2 + 1}{k^2 - 1} \right) - \sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

$$Z_b = \frac{Z_L}{2} \left( \frac{k^2 + 1}{k^2 - 1} \right) - \sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

$$Z_c = 2\sqrt{Z_s Z_L} \left( \frac{k}{k^2 - 1} \right)$$

 $\pi$  - NETWORK


$$Z_a = Z_s \left( \frac{k^2 - 1}{k^2 - 2sk + 1} \right)$$

$$Z_b = \frac{\sqrt{Z_s Z_L}}{2} \left( \frac{k^2 - 1}{k} \right)$$

$$Z_c = Z_L \left( \frac{k^2 - 1}{k^2 - 2\frac{k}{s} + 1} \right)$$

O - NETWORK



$$Z_a = Z_s \left( \frac{k^2 - 1}{k^2 - 2sk + 1} \right)$$

$$Z_b = \frac{\sqrt{Z_s Z_L}}{4} \left( \frac{k^2 - 1}{k} \right)$$

$$Z_c = Z_L \left( \frac{k^2 - 1}{k^2 - 2\frac{k}{s} + 1} \right)$$

**Fig. 40.** Diagrams and mathematics of the most frequently used attenuator networks.

design of an attenuator is tremendously simplified by means of the tables since all that need be done is merely to substitute the figures given in the appropriate places, and solve the resulting expression.

#### Illustrative

**Problem 21:** A T-network having an attenuation of 15 db is required between a source impedance of 1000 ohms and a load impedance of 500 ohms. Calculate  $Z_a$ ,  $Z_b$ , and  $Z_c$ .

**Solution:** Solve first for  $Z_a$ . The tabular values for  $(k^2 + 1/k^2 - 1)$  and  $(k/k^2 - 1)$  are obtained for 15 db attenuation as 1.0653 and 0.18363 respectively by reading off in the proper column alongside  $n(\text{db}) = 15$ . For the benefit of

the reader who wishes to follow the calculations through on an ordinary slide rule, the substitutions are shown below in only three significant figures. (These equations appear next to the T-network in Fig. 40.)

$$Z_a = (1000) (1.07) - 2 \sqrt{1000} \times 500 \quad (0.184)$$

$$Z_a = 1070 - 2 (707) \quad (0.184)$$

$$Z_a = 1070 - 260 = 810 \text{ ohms}$$

$$Z_b = (500) (1.07) - 260$$

$$Z_b = 535 - 260$$

$$Z_b = 275 \text{ ohms}$$

$$Z_c = 260 \text{ ohms (obtained in both previous computations)}$$

Other attenuator networks are handled in an identical fashion, substituting wherever possible the pre-calculated tabular values for various expressions involving  $k$ .

Two additional facts should be noted: the reciprocal of  $k$  ( $1/k$ ) appears in many of the network equations. This is symbolized by "r" in the table. In addition, the symbol "S" appears in several places; this symbol represents the function  $\sqrt{Z_s/Z_L}$  and cannot be given tabular values because it does depend upon the source and load impedances which vary from case to case.

### 13. Reactive Networks

Reactive networks may have their elements arranged in much the same circuitry as the resistive networks that have been described. It should be remembered that reactive networks have responses that are sensitive to frequency since impedances change with frequency. The attenuation also changes with frequency.

A valuable use of capacitors as a voltage-divider arrangement is shown in Fig. 41.

In this circuit the primary and secondary coils are tuned with capacitors. Usually the coupling is slight so that the  $Q$  of the cir-

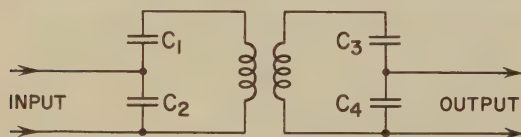


Fig. 41. A capacitor voltage divider.

cuit is held very high. The voltage-divider arrangement of the capacitors permits sending energy at a low impedance level into the tuned circuits. As the  $Q$  is disturbed very little the tuned circuit can be made to have a high degree of frequency discrimina-

tion. The output is also arranged to prevent loading the tuned circuit with the connecting circuit.

A similar arrangement is made with the coil tapped with the configuration of an autotransformer. In this way the tuned circuit is left free to select a narrow band of frequencies of a single frequency.

### Review Questions

1. What are the common types of impedance-matching devices?
2. Of what use is a logarithm?
3. Describe a decibel.
4. How are transformer windings arranged to give series-aiding connections?
5. What is mutual inductance?
6. Give the relationship between turns, voltage, current, and power for a two winding transformer.
7. What is the phase relationship between the primary and secondary voltages?
8. Describe an autotransformer.
9. What is meant by maximum power transfer?
10. Under what conditions is maximum power transfer possible?
11. Does the secondary load affect the impedance that a transformer presents to a source?
12. What are the various causes of losses in a transformer?
13. Draw an equivalent network of a transformer.
14. What is reflected impedance in a transformer?
15. Can a transformer always be used to increase the power delivered to a load?
16. What is the input impedance of a cathode follower?
17. How much gain is obtained in a cathode follower circuit?
18. Show how to compute the output impedance of a cathode follower.
19. Of what use are resistance networks?
20. What is image impedance? Iterative impedance?
21. Describe a four-terminal network.
22. Construct a continuity-bridge network of 16 db loss.

$n$ (db)	$r = \frac{1}{k}$	$k$	$k^2$	$\frac{k-1}{k+1}$	$\frac{k+1}{k-1}$	$\frac{k}{k^2-1}$	$\frac{k^2-1}{k} = k-r$	$\frac{k^2+1}{k^2-1}$	$\frac{k-1}{k} = 1-r$	$\frac{k}{k-1} = \frac{1}{1-r}$	$\frac{1}{k-1}$	$n$ (db)
0.05	0.994260	1.0057731	1.011579	0.0028783	347.43	86.8618	0.011513	173.73	0.0057395	174.22	173.22	0.05
0.1	0.988555	1.011579	1.023292	0.0057562	173.73	43.4303	0.023029	86.866	0.011448	87.363	86.363	0.1
0.2	0.97724	1.023292	1.047128	0.011512	86.866	21.713	0.046052	43.933	0.022762	43.933	42.933	0.2
0.3	0.96605	1.035143	1.071520	0.017268	57.911	14.473	0.069093	28.965	0.039499	29.455	28.455	0.3
0.4	0.95499	1.047128	1.096477	0.023022	43.437	10.854	0.092138	21.730	0.045007	22.219	21.219	0.4
0.5	0.94406	1.059254	1.12202	0.028774	34.754	8.6810	0.11519	17.391	0.055939	17.877	16.877	0.5
0.6	0.93325	1.071520	1.14815	0.034525	28.965	7.2327	0.13827	14.499	0.068746	14.982	13.982	0.6
0.7	0.92257	1.083928	1.17490	0.040274	24.830	6.1974	0.16136	12.435	0.077438	12.915	11.915	0.7
0.8	0.91201	1.096477	1.20227	0.046019	21.730	5.4269	0.18447	10.888	0.087988	11.365	10.365	0.8
0.9	0.90157	1.10917	1.23027	0.051763	19.319	4.8168	0.20760	9.6853	0.098429	10.160	9.1600	0.9
1.0	0.89125	1.12202	1.25893	0.057502	17.391	4.3335	0.23077	8.7237	0.10875	9.1954	8.1954	1.0
1.1	0.88105	1.13931	1.28825	0.063237	15.814	3.9376	0.25396	7.9384	0.11895	8.4069	7.4069	1.1
1.2	0.87096	1.14815	1.31826	0.068968	14.499	3.6076	0.27719	7.2842	0.12904	7.7499	6.7499	1.2
1.3	0.86099	1.16145	1.34896	0.074695	13.398	3.3283	0.30046	6.7313	0.13901	7.1939	6.1939	1.3
1.4	0.85114	1.17490	1.38038	0.080418	12.435	3.0888	0.32376	6.2579	0.14886	6.7176	5.7176	1.4
1.5	0.84139	1.18850	1.41254	0.086132	11.610	2.8909	0.33711	5.8480	0.15861	6.3050	5.3050	1.5
1.6	0.83176	1.20227	1.44544	0.091846	10.888	2.6991	0.37051	5.4899	0.16824	5.9439	4.9439	1.6
1.7	0.82224	1.21618	1.47911	0.097551	10.251	2.5384	0.39394	5.1744	0.17776	5.6358	4.6358	1.7
1.8	0.81283	1.23027	1.51356	0.103249	9.6853	2.3956	0.41744	4.8944	0.18717	5.3427	4.3427	1.8
1.9	0.80353	1.24452	1.54882	0.108939	9.1794	2.2676	0.44099	4.6442	0.19447	5.0897	4.0897	1.9
2.0	0.79433	1.25893	1.58489	0.11463	8.7241	2.1523	0.46460	4.4195	0.20567	4.8620	3.8620	2.0
2.2	0.77625	1.28825	1.65059	0.12507	7.9384	1.9531	0.51200	4.0322	0.22375	4.4692	3.4692	2.2
2.4	0.75858	1.31826	1.72780	0.13728	7.2842	1.7867	0.55968	3.7108	0.24142	4.1421	3.1421	2.4
2.5	0.74989	1.33252	1.77828	0.14293	6.9966	1.7133	0.58363	3.5698	0.25011	3.9983	2.9983	2.5
2.6	0.74131	1.34896	1.81970	0.14856	6.7313	1.6457	0.60765	3.4399	0.25829	3.8657	2.8657	2.6
2.8	0.72444	1.38038	1.90546	0.15980	6.2579	1.5245	0.65594	3.2088	0.27556	3.6289	2.6289	2.8
3.0	0.70795	1.41254	1.99526	0.17100	5.8480	1.4192	0.70459	3.0065	0.29205	3.4240	2.4240	3.0
3.2	0.69183	1.44544	2.08930	0.18215	5.4899	1.3269	0.75361	2.8360	0.30817	3.2450	2.2450	3.2
3.4	0.67608	1.47911	2.18776	0.19326	5.1744	1.2453	0.80302	2.6838	0.32392	3.0872	2.0872	3.4
3.5	0.66854	1.49623	2.2387	0.19879	5.0304	1.2079	0.82789	2.6147	0.33166	3.0152	2.0152	3.5

3.6	0.60663	1.51356	2.2909	0.20432	4.8944	1.1725	0.85289	2.5493	0.33931	2.9472	1.9472	3.6
3.8	0.64565	1.58482	2.3988	0.21532	4.6442	1.1072	0.90314	2.4298	0.35435	2.8221	1.8221	3.8
4.0	0.69096	1.58489	2.5519	0.22627	4.4194	1.0483	0.95393	2.3229	0.36904	2.7097	1.7097	4.0
4.5	0.90666	1.67880	2.8184	0.25340	3.9464	0.92323	1.08314	2.0989	0.40434	2.4732	1.4732	4.5
5.0	0.56234	1.77828	3.1623	0.28013	3.5698	0.82241	1.21594	1.9249	0.43766	2.2849	1.2849	5.0
5.5	0.53088	1.88365	3.5481	0.30643	3.2633	0.73922	1.35277	1.7849	0.46912	2.1317	1.1371	5.5
6.0	0.50119	1.99526	3.9811	0.33228	3.0095	0.66932	1.49407	1.6709	0.49881	2.0048	1.0048	6.0
6.5	0.47315	2.1135	4.4668	0.35764	2.7951	0.60964	1.6403	1.5769	0.52685	1.8907	0.8907	6.5
7.0	0.44666	2.2387	5.0119	0.38246	2.6146	0.55801	1.7920	1.4985	0.55332	1.80730	0.80730	7.0
7.5	0.42170	2.3714	5.6234	0.40677	2.4584	0.51291	1.9197	1.4326	0.57830	1.72918	0.72918	7.5
8.0	0.39811	2.5119	6.3096	0.43051	2.3228	0.47309	2.1138	1.3767	0.60189	1.66142	0.66142	8.0
8.5	0.37584	2.6607	7.0795	0.45366	2.2043	0.43765	2.2849	1.3290	0.62416	1.60216	0.60216	8.5
9.0	0.35481	2.8184	7.9433	0.47622	2.0959	0.40592	2.4636	1.2880	0.64519	1.54993	0.54993	9.0
9.5	0.33497	2.9854	8.9125	0.49817	2.0074	0.37730	2.6504	1.2528	0.66503	1.50368	0.50368	9.5
10.0	0.31623	3.1623	10.000	0.51950	1.9249	0.35137	2.8561	1.2222	0.68377	1.46247	0.46247	10.0
10.5	0.29854	3.3497	11.220	0.54020	1.8512	0.32775	3.0512	1.1957	0.70146	1.42559	0.42559	10.5
11.0	0.28184	3.5481	12.589	0.56026	1.7849	0.30616	3.2663	1.1726	0.71816	1.39245	0.39245	11.0
11.5	0.26607	3.7584	14.125	0.57969	1.7251	0.28635	3.4923	1.1524	0.73393	1.36253	0.36253	11.5
12.0	0.25119	3.9811	15.849	0.59848	1.6709	0.26811	3.7299	1.1347	0.74881	1.33545	0.33545	12.0
12.5	0.23714	4.2170	17.783	0.61664	1.6217	0.25127	3.9799	1.1192	0.76286	1.31085	0.31085	12.5
13.0	0.22387	4.4668	19.953	0.63416	1.5769	0.23568	4.2429	1.1055	0.77613	1.28845	0.28845	13.0
13.5	0.21135	4.7315	22.387	0.65105	1.5360	0.22123	4.5202	1.0935	0.78665	1.26799	0.26799	13.5
14.0	0.19953	5.0119	25.119	0.66733	1.4985	0.20780	4.8124	1.0829	0.80047	1.24926	0.24926	14.0
14.5	0.18836	5.3088	28.184	0.68298	1.4642	0.19529	5.1204	1.0736	0.81164	1.23208	0.23208	14.5
15.0	0.17783	5.6234	31.623	0.69804	1.4326	0.18363	5.4456	1.0653	0.82217	1.21629	0.21629	15.0
15.5	0.16788	5.9566	35.481	0.71250	1.4035	0.17275	5.7887	1.0580	0.83212	1.20175	0.20175	15.5
16.0	0.15849	6.3096	39.811	0.72639	1.3767	0.16257	6.1511	1.0515	0.84151	1.18834	0.18834	16.0
16.5	0.14962	6.6834	44.668	0.73970	1.3519	0.15305	6.5338	1.0458	0.85038	1.17595	0.17595	16.5
17.0	0.14125	7.0795	50.119	0.75246	1.3290	0.14413	6.9382	1.04071	0.85875	1.16449	0.16449	17.0
17.5	0.13335	7.4989	56.234	0.76468	1.3077	0.13577	7.3655	1.03621	0.86665	1.15387	0.15387	17.5
18.0	0.12589	7.9433	63.096	0.77637	1.2880	0.12792	7.8174	1.03220	0.87411	1.14402	0.14402	18.0
18.5	0.118850	8.4139	70.795	0.78755	1.2698	0.12055	8.2950	1.02866	0.88151	1.13488	0.13488	18.5
19.0	0.112202	8.9125	79.423	0.79823	1.2528	0.11363	8.8003	1.02550	0.88780	1.12638	0.12638	19.0
19.5	0.105925	9.4406	89.125	0.80844	1.2369	0.10713	9.3347	1.02269	0.89407	1.11847	0.11847	19.5
20.0	0.100000	10.0000	100.000	0.81818	1.2222	0.10101	9.9000	1.02020	0.90000	1.11111	0.11111	20.0



## IMPEDANCE MATCHING

$n$ (db)	$\tau = \frac{1}{k}$	$k$	$k^2$	$\frac{k-1}{k+1}$	$\frac{k}{k^2-1}$	$\frac{k^2-1}{k}$	$\frac{k^2+1}{k^2-1}$	$\frac{k-1}{k} = 1-r$	$\frac{k}{k-1} = \frac{1}{1-r}$	$\frac{1}{k-1}$	$n$ (db)
20.5	0.094406	10.5925	112.202	0.82747	1.2085	0.085255	1.01799	0.90559	1.10425	0.10425	20.5
21.0	0.089125	11.2202	125.893	0.83634	1.1957	0.089841	1.01601	0.91087	1.09875	0.097845	21.0
21.5	0.084139	11.8850	141.254	0.84478	1.1837	0.084739	1.01426	0.91586	1.09187	0.091870	21.5
22.0	0.079433	12.589	158.49	0.85282	1.1726	0.079935	1.01270	0.92057	1.08629	0.086291	22.0
22.5	0.074989	13.335	177.83	0.86048	1.1621	0.075411	1.01126	0.92501	1.08107	0.081070	22.5
23.0	0.070795	14.125	199.53	0.86777	1.1524	0.071148	1.01007	0.92921	1.07619	0.076190	23.0
23.5	0.066834	14.962	223.77	0.87470	1.1432	0.067133	1.00897	0.93317	1.07162	0.071623	23.5
24.0	0.063096	15.789	251.19	0.88130	1.1347	0.063348	1.00799	0.93690	1.06734	0.067345	24.0
24.5	0.059566	16.788	281.84	0.88756	1.1267	0.059778	1.00712	0.94043	1.06334	0.063339	24.5
25.0	0.056234	17.783	316.23	0.89352	1.1192	0.056413	1.00634	0.94377	1.05958	0.059584	25.0
25.5	0.053088	18.836	354.81	0.89917	1.1121	0.053238	1.00565	0.94691	1.05607	0.056066	25.5
26.0	0.050119	19.953	398.11	0.90455	1.1055	0.050246	1.00504	0.94988	1.05276	0.052762	26.0
26.5	0.047315	21.135	446.68	0.90965	1.0993	0.047422	1.00449	0.95268	1.04966	0.049665	26.5
27.0	0.044668	22.387	501.19	0.91448	1.0935	0.044757	1.00400	0.95533	1.04676	0.046757	27.0
27.5	0.042170	23.714	562.34	0.91907	1.0881	0.042245	1.00356	0.95783	1.04403	0.044026	27.5
28.0	0.039811	25.119	630.96	0.92343	1.0829	0.039874	1.00317	0.96019	1.04146	0.041461	28.0
28.5	0.037584	26.607	707.95	0.92755	1.0781	0.037636	1.00283	0.96242	1.03905	0.039052	28.5
29.0	0.035481	28.184	794.33	0.93147	1.0736	0.035526	1.00252	0.96452	1.03679	0.036786	29.0
29.5	0.033497	29.854	891.25	0.93518	1.0693	0.033534	1.00225	0.96650	1.03466	0.034657	29.5
30.0	0.031623	31.623	1,000.0	0.93869	1.0653	0.031655	1.00200	0.96836	1.03266	0.032635	30.0
31.0	0.028184	35.481	1,258.9	0.94518	1.0580	0.028207	1.00159	0.97182	1.02900	0.029001	31.0
31.5	0.026607	37.584	1,412.5	0.94817	1.0547	0.026627	1.00142	0.97339	1.027334	0.027334	31.5
32.0	0.025119	39.811	1,584.9	0.95099	1.0515	0.025135	1.00126	0.97488	1.02577	0.025765	32.0
33.0	0.022387	44.668	1,995.3	0.95621	1.0458	0.022398	1.00100	0.97761	1.02290	0.022900	33.0
34.0	0.019953	50.119	2,511.9	0.96088	1.04072	0.019961	1.00080	0.98005	1.02036	0.020359	34.0
34.5	0.018836	53.088	2,818.4	0.96302	1.03840	0.018843	1.00071	0.98116	1.01920	0.019198	34.5
35.0	0.017783	56.234	3,162.3	0.96506	1.03621	0.017788	1.00063	0.98222	1.01810	0.018105	35.0
36.0	0.015849	63.096	3,981.1	0.96880	1.03221	0.015853	1.00050	0.98415	1.01610	0.016104	36.0
37.0	0.014125	70.795	5,011.9	0.97214	1.02866	0.014128	1.00040	0.98588	1.01433	0.014328	37.0
37.5	0.013335	74.989	5,623.4	0.97368	1.02703	0.013338	1.00036	0.98666	1.01352	0.013516	37.5

38.0	0.012589	79.433	0.97513	1.02550	0.012591	79.430	1.00032	0.98741	1.01275	0.012750	38.0
39.0	0.012202	89.125	0.97781	1.02270	0.0112216	89.114	1.00025	0.98878	1.01135	0.011348	39.0
40.0	0.010000	100.000	0.98020	1.02020	0.0100010	99.990	1.00020	0.99000	1.01010	0.010101	40.0
40.5	0.0094406	105.925	0.98130	1.01906	0.0094414	105.916	1.00018	0.99056	1.00953	0.0095306	40.5
41.0	0.0089125	112.202	0.98233	1.01799	0.0089134	112.193	1.00016	0.99109	1.00899	0.0089926	41.0
42.0	0.0079433	125.89	0.98424	1.01601	0.0079436	125.88	1.00013	0.99206	1.00800	0.0080070	42.0
43.0	0.0070795	141.25	0.98594	1.01426	0.0070795	141.24	1.00010	0.99292	1.00713	0.0071301	43.0
43.5	0.0066834	149.62	0.98746	1.01346	0.0066834	149.61	1.00009	0.99332	1.00673	0.0067286	43.5
44.0	0.0063096	158.49	0.98876	1.01270	0.0063096	158.49	1.00008	0.99369	1.00635	0.0063496	44.0
45.0	0.0056234	177.83	0.98887	1.01131	0.0056234	177.83	1.00005	0.99438	1.00566	0.0056551	45.0
46.0	0.0050119	199.53	0.99003	1.01007	0.0050119	199.53	1.00005	0.99499	1.00504	0.0050370	46.0
46.5	0.0047315	211.35	0.99058	1.00951	0.0047315	211.35	1.000045	0.99527	1.00475	0.0047540	46.5
47.0	0.0044608	223.87	0.99111	1.00897	0.0044608	223.87	1.000040	0.99553	1.00449	0.0044869	47.0
48.0	0.0039811	251.19	0.99207	1.00799	0.0039811	251.19	1.000032	0.99602	1.00400	0.0039970	48.0
49.0	0.0035481	281.84	0.99293	1.00712	0.0035481	281.84	1.000025	0.99645	1.00356	0.0035607	49.0
50.0	0.0031623	316.23	0.99370	1.00634	0.0031623	316.23	1.000020	0.99684	1.00317	0.0031723	50.0
51.0	0.0028184	354.81	0.99438	1.00565	0.0028184	354.81	1.000016	0.99718	1.00283	0.0028204	51.0
52.0	0.0025119	398.11	0.99499	1.00504	0.0025119	398.11	1.000013	0.99749	1.00252	0.0025182	52.0
54.0	0.0019953	501.19	0.99602	1.00400	0.0019953	501.19	1.000008	0.99801	1.00200	0.0019992	54.0
55.0	0.0017783	562.34	0.99645	1.00356	0.0017783	562.34	1.000006	0.99822	1.00178	0.0017815	55.0
56.0	0.0015849	630.96	0.99684	1.00317	0.0015849	630.96	1.000005	0.99842	1.00159	0.0015874	56.0
57.0	0.0014125	707.95	0.99718	1.00283	0.0014125	707.95	1.000004	0.99859	1.00141	0.0014145	57.0
58.0	0.0012589	794.33	0.99749	1.00252	0.0012589	794.33	1.000003	0.99874	1.00126	0.0012605	58.0
60.0	0.0010000	1000.0	0.99800	1.00200	0.0010000	1000.0	1.000002	0.99900	1.00100	0.0010010	60.0
65.0	0.00056234	$3.1623 \times 10^6$	0.99888	1.00112	0.00056234	1.778.3	1.000001	0.99944	1.00056	0.00056265	65.0
70.0	0.00031623	$3.1623 \times 10^7$	0.99937	1.00063	0.00031623	3.162.3	1.000000	0.99968	1.00032	0.00031633	70.0
75.0	0.00017783	$5.623.4 \times 10^8$	0.99964	1.00036	0.00017783	5.623.4	1.000000	0.99982	1.00018	0.00017786	75.0
80.0	0.00010000	$10.000 \times 10^8$	0.99980	1.00020	0.00010000	10.000	1.000000	0.99990	1.00010	0.00010001	80.0
85.0	0.000056234	$17.783. \times 10^8$	0.99989	1.00011	0.000056234	17.783.	1.000000	0.99994	1.00006	0.000056337	85.0
90.0	0.000031623	$31.623. \times 10^9$	0.99994	1.00006	0.000031623	31.623.	1.000000	0.99997	1.00003	0.000031624	90.0
95.0	0.000017783	$56.234. \times 10^9$	0.99996	1.00004	0.000017783	56.234.	1.000000	0.99998	1.00002	0.000017783	95.0
100.0	0.000010000	$10^{10}$	0.99998	1.00002	0.000010000	$10^{10}$	1.000000	0.99999	1.00001	0.000010000	100.0

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## Chapter 3

### IMPEDANCE MATCHING AT AUDIO FREQUENCIES

#### 14. General Requirements

Impedance matching at audio frequencies is important because the ear is reasonably sensitive to frequency distortion. It has been shown that source and load impedances should be matched to conserve power. However, it is not always possible in circuits involving capacitance and inductance to design components to prevent frequency discrimination since C and L elements are both frequency sensitive. Some of the problems of impedance matching in audio circuit arrangements are considered here.

#### 15. Output Tube to Speaker

There are two basic types of amplifiers — voltage and power. The output tube to a complete amplifier system is usually a power amplifier inasmuch as it must deliver enough power to drive the loudspeaker.

It might be well to review a few fundamentals in viewing this problem of impedance matching. When an alternating signal voltage is impressed on the control grid of a power tube, the power tube may be considered as a generator having a voltage of  $\mu E_g$  and an internal resistance of  $r_p$ . If a load resistance  $R_L$  is connected to the tube, the current flow  $I_p = \mu E_g / (r_p + R_L)$ . The power delivered to the load will be:

$$\text{watts} = I_p^2 R_L = \left( \frac{\mu E_g}{r_p + R_L} \right)^2 R_L \quad (138)$$

For maximum power transfer the load impedance must equal the source impedance, or  $r_p$  must equal  $R_L$ . However, in power amplification, signal distortion must be considered. It is necessary to view the problem not only from the shape of the  $I_p$ - $E_g$  curve but also from the circuit as a whole.

There are several types of amplifiers. In one the  $r_p$  must equal  $R_L$ , in the triode amplifier the  $R_L$  must equal twice the  $r_p$ , there is also a pentode or beam-power type where  $r_p$  equals five to ten times  $R_L$ , the bridging type where  $r_p$  equals one tenth  $R_L$ , and the case of input to an amplifier where the load is infinite.

Any tube manual suggests the output impedance that is best for the tube and the voltages that are listed. These may range from 1000 ohms to more than 20,000 ohms, or the load might be a loud speaker with impedances of as little as four ohms.

Using an example of a triode tube where the load should be twice the tube resistance, we can draw an example such as Fig. 42.

The relationship between turns, voltage, current, and impedance of a transformer may be written:

$$\frac{T_1}{T_2} = \frac{E_1}{E_2} = \frac{I_2}{I_1} = \sqrt{\frac{Z_1}{Z_2}} \quad (139)$$

In the case of the circuit of Fig. 42, the turns ratio may be computed:

$$\frac{T_1}{T_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{2500}{4}} = 25 \quad (140)$$

Another problem we encounter quite frequently is matching a beam power pentode to a loudspeaker. A popular type is the 6V6 tube. The tube manual tells us that with a supply of 250 volts the tube has an  $r_p$  of 5200  $\Omega$  but for best frequency response requires a load resistance of 5000  $\Omega$ . We want to match this tube to a speaker having 8  $\Omega$  impedance.

$$\sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{5000}{8}} = 25$$

The transformer should have a turns ratio of 25:1.

The amount of power that can be handled by a transformer is determined by the current and voltage ratings of the windings and the allowable losses. The primary frequently contains a d-c component, the d-c plate current, that limits the inductance and frequency response. The voltage induced in the winding of a transformer is:

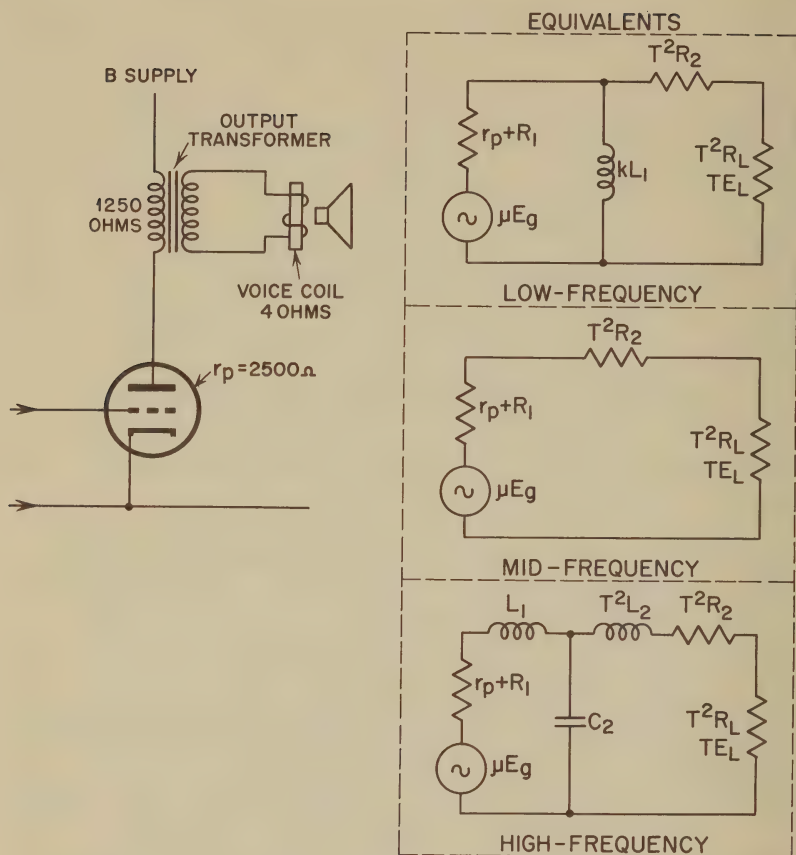


Fig. 42. A triode output tube and its equivalent circuits.

E induced =

$$(4.4) \frac{(\text{frequency}) (\text{turns}) (\text{flux-density}) (\text{cross-section of core})}{10^8} \quad (141)$$

where E is in volts, frequency is in cycles, flux-density is in gaussses, and cross-section is in  $\text{cm}^2$ .

This formula says that the induced voltage in an individual transformer is proportional to the product of the frequency and the flux density. At low frequency the flux density is high and more distortion is introduced because of the saturation of the iron. The maximum allowable flux density is determined by the maximum allowable distortion.

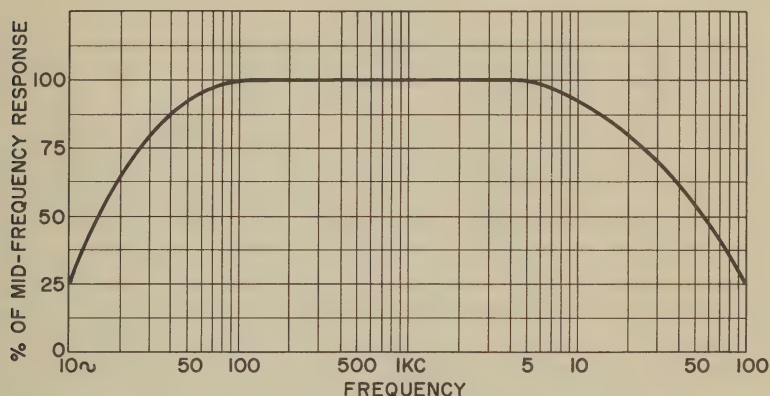
The output transformer causes a reduction in the output at both



low and high frequencies. The reduced power at the low frequencies results from the shunting action of the transformer primary inductance on the load. This is shown in Fig. 42, where the low-frequency equivalent is  $kL_1$ .

The middle frequencies are amplified without reactance as shown in equivalent circuit of Fig. 42 because neither the shunting effect of the coupling coefficient times primary incremental inductance nor the shunting effect of capacity in the winding appreciably affects the gain. In other words, the primary inductance at mid-frequencies is great enough to have its shunting effect discounted, and the leakage reactances are low enough to be neglected.

The reduced output at high frequencies is caused by the loss in voltage through the leakage reactances, a result of load current and capacitive current due to shunting capacitance. In the high-



**Fig. 43.** Graph showing a universal frequency-response curve.

frequency equivalent circuit of Fig. 42, where the leakage reactance is shown as  $L_1$  and  $T^2L_2$ , the capacitive current flows through unit  $C_2$ . As the result of these losses the response curve indicates reduced level at high frequencies (Fig. 43).

In order to extend the flat response of the curve of Fig. 43 into the high-frequency region, the leakage inductance should be low. Some improvement may be made for a given triode and transformer in the high-frequency portion of the response without disturbing the low-frequency response materially, by increasing the resistance of the load. This is not always practical.

The low-frequency portion of the response curve can be made flatter with a wider range by having the primary impedance of the

transformer high. The greater flux density and tendency to saturate the iron core is thus taken care of at the low frequencies. This too must be done carefully so that the capacitance shunt at high frequencies is not also increased.

The voltage at the load at various frequencies may be determined from the equivalent circuits of Fig. 42. At the mid-frequency range the computations may be made from:

$$TE_L = \frac{\mu E_g T^2 R_L}{(r_p + R_1) + T^2 (R_2 + R_L)} \quad (142)$$

For the low- and high-frequency ranges it is necessary to determine the values of leakage inductance and shunt capacitance.

## 16. Tube-to-Line Speaker

In the typical job of matching an amplifier tube to a number of speakers, the problem is much greater, and a number of fundamental questions arise.

First, the quality of the amplifier output must be determined. For example, consider background music in a restaurant: when music levels are low, as they are in background music, the ear response at both low and high frequencies is poor. It would serve little purpose to transmit quality music over a system in which the volume is so low that the low and high regions of music can not be heard. Almost any wiring arrangement of the output is satisfactory for this type of installation.

However, where the loudspeakers are used to give reproduction near the level of the original volume, as in a movie theater or lecture hall, quality is important. It then becomes necessary to reproduce the band of frequencies needed. A system used purely for music requires a much broader frequency response than a system used to reproduce voice.

A problem in connecting several loudspeakers is given in Fig. 44, where the output of an amplifier is fed to three speakers.

The problem of Fig. 44 shows three loudspeakers fed from one triode-amplifier output transformer. The speakers may be assumed to have sufficiently high impedance to match the impedance of the line that feeds them. This is an important point, because not only does quality enter into this problem, but so does the transfer of power. Maximum power transfer (MPT) occurs to each speaker when:

$$Z_s = Z_L = (R_s + jX_s) = (R_L + jX_L) \quad (143)$$

This would be simple if the speakers could be bought with a large range of impedance values. But this is not the only problem, for a line is not merely inductive in reactance, but also in capacitance. To match impedances, therefore, it is necessary first to match impedance magnitudes and then to use some means of correcting the angle of the two impedances that are to be matched.

At the transformer the impedance looking in toward the tube

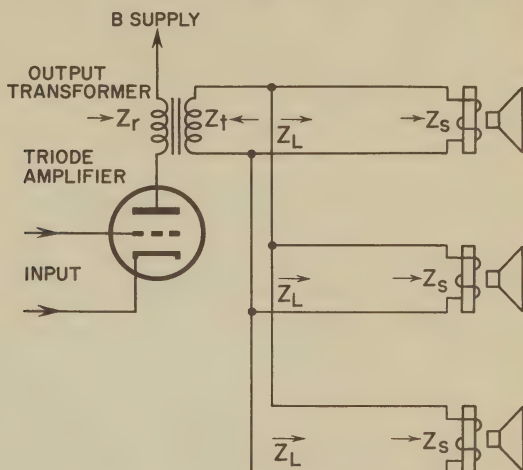


Fig. 44. Schematic of one triode supplying three loudspeakers.

$Z_t$  must face  $Z_L/3$ . Again the angle and magnitudes of these two impedances must be matched.

At the tube, the impedance of the load as seen through the transformer that is  $Z_r$  must fit the requirement of the triode (the load impedance must be twice the tube impedance).

The matching of these transformers, lines, and speakers must also be done at low-, mid-, and high-frequency ranges. The limit of human hearing response simplifies the problem considerably. Most people do not notice the absence of frequencies below a number even as high as 500 cycles, although some like to hear down to 200 cycles. Measurements of a system need not be made all the way down to power frequencies, but satisfactory results on a practical level can be obtained if measurements stop some place between 200 and 500 cycles.

As for the high end of the frequency band, measurements might stop at a frequency that may seem particularly low. Good reproduction occurs with responses up to 5000 cycles. In fact most A-M radio stations, as heard on average radio receivers, do not encompass more than 3500 cycles. The stations transmit frequencies of

ranges higher than 3500 cycles but the ordinary radio receiver does not respond to them. Small portable sets are likely to have responses even lower than 3500 cycles, down to as low as 1500 or 2000 cycles. Comparatively, reproduction of sound frequencies from 5000 cycles to 8000 cycles is extremely good, and adequate for all but high fidelity application. To provide responses up to 15,000 cycles is rarely necessary. Not only are most adults unable to hear the higher ranges, but the acoustics to handle these extreme ranges would have to be very carefully treated.

It would therefore be sufficient to make measurements on the system from the input of the last amplifier to the loudspeaker input at frequencies from 400 to 5000 cycles.

### 17. Use of Multimatch Transformers

Many tube-to-speaker transformers are provided with a large number of taps on the secondary winding. This arrangement permits proper impedance matching between the given output tube

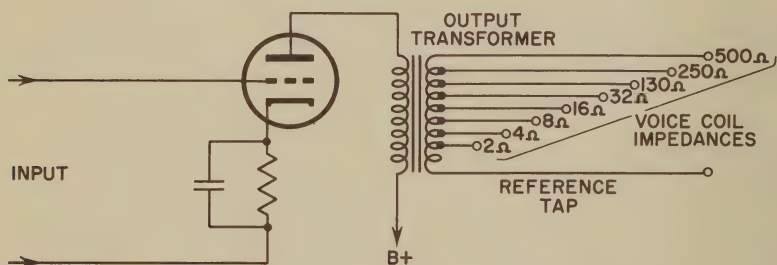


Fig. 45. Application of a universal or multimatch transformer.

and individual speakers having a variety of voice-coil impedances. A typical universal or multimatch transformer of this kind is shown schematically in Fig. 45. The turns ratio and tap positions are chosen to provide the source impedances shown to properly match any of the more common voice-coil impedances or to match a transmission line of 250 or 500 ohms.

Universal transformers are usually designed so that only one tap may be used at one time. If the speaker voice coil has one of the standard impedances (4, 8, 16, or 32 ohms) it may be connected directly across the corresponding tap and the reference tap. When the impedance of the voice coil does not exactly match the tap ratings, good results will still be obtained if the voice coil is con-



nected across the tap having the nearest impedance provided that the difference does not exceed 10%. If, at even this proximity, a match proves impossible, the tap immediately lower than the voice coil impedance should be used. Although following this rule results in a smaller power transfer, fidelity will not suffer to any great extent.

In addition to the source impedances available at the taps shown in Fig. 45, others may be obtained by various combinations not involving the reference leg. For example, the 32-ohm tap may be used in conjunction with the 130 tap immediately above it to arrive at a source impedance different from any of the others. This impedance is not, however, the simple arithmetic difference between the two taps; that is, the impedance between the 32-ohm tap and the 130-ohm tap is *not* 98 ohms. To find the impedance between any two taps, the following equation may be used:

$$Z_x = Z_a \left( \sqrt{\frac{Z_b}{Z_a}} - 1 \right)^2 \quad (143A)$$

Thus, substituting 32 ohms for  $Z_a$  (lower impedance) and 130 ohms for  $Z_b$  (higher impedance),  $Z_x$  (the impedance between these two taps) may be determined as follows:

$$Z_x = 32 \left( \sqrt{\frac{130}{32}} - 1 \right)^2$$

$$Z_x = 32.6 \text{ ohms}$$

#### Illustrative

**Problem 22:** A 20-ohm voice coil is to be matched to the output transformer of Fig. 45. Determine the taps between which this voice coil should be connected.

**Solution:** An impedance close to 20 ohms may be found between two of the higher impedance taps. Substitutions are made in the equation until a close approximation is obtained. Thus, if the 130-ohm and 250-ohm impedances are substituted we obtain:

$$Z_x = 130 \left( \sqrt{\frac{250}{130}} - 1 \right)^2$$

$$Z_x = 130 \times 0.15$$

$$Z_x = 19.5 \text{ ohms}$$

This is sufficiently close to the desired value of 20 ohms.

## 18. Matching Multiple Speaker Installations

Multiple speaker installations are found in automobile radio systems, public address systems, and often in high-fidelity repro-



duction equipment. For maximum transfer of power with minimum distortion, it is important that an impedance match be maintained between the output transformer and the group of speakers. As with other electrical loads, speakers may be connected in series, parallel, or series-parallel. The type of connection used will be determined by one of the following considerations:

(a) Two or more speakers in series present a load impedance equal to the sum of their individual impedances.

(b) Two or more speakers in parallel, each having the same voice coil impedance, present a total impedance equal to the impedance of any one voice coil divided by the number of speakers.

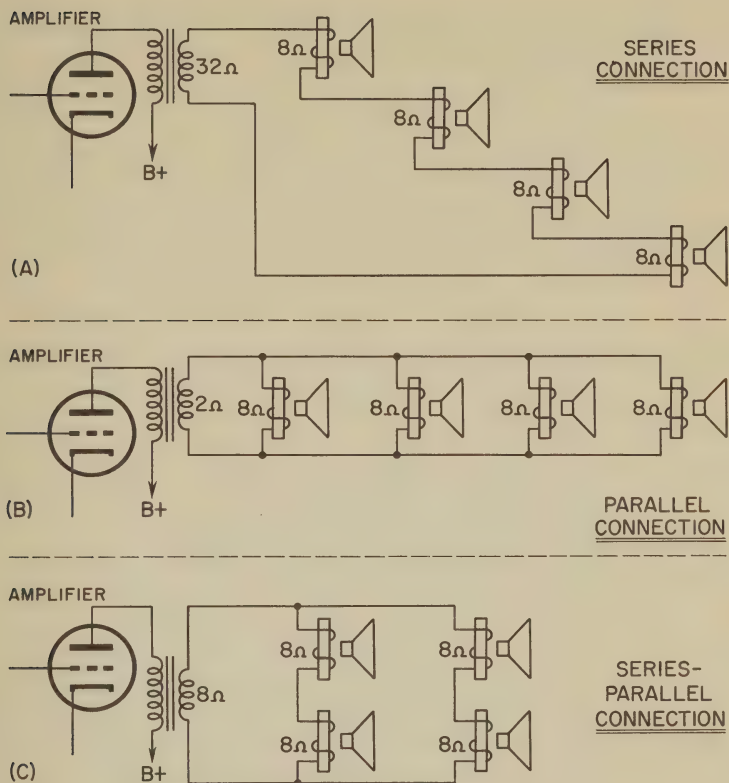
(c) Four or more speakers in series-parallel, each having the same voice-coil impedance, present a total impedance equal to the impedance of any series branch divided by the number of such series lines connected in parallel. (This assumes equal total impedance of each series branch.)

Figure 46 illustrates several typical multiple speaker installations using these laws.

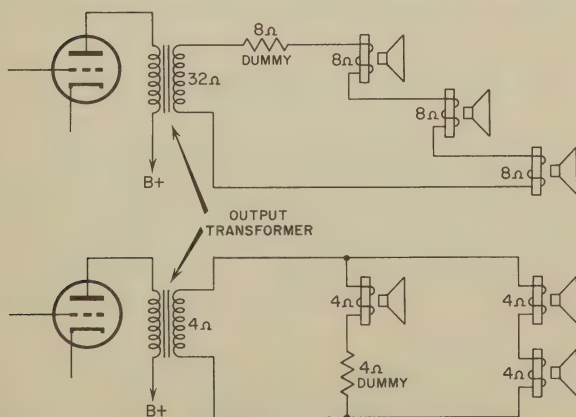
As it so often happens, the resolution of a problem in multiple speaker installation may be difficult because of an incorrect number of speakers for any of the connections or because the output transformer cannot be made to match series, parallel, or series-parallel hookups. Such cases are often handled by insertion of a dummy load in place of a "missing" speaker or by changing the total load impedance of the available combination. For example, suppose we have three 8-ohm voice coils we want to match to a transformer whose output impedance is 32 ohms. Correction for the mismatch is made by inserting a dummy load of 8 ohms and by connecting all components in series as shown in Fig. 47.

Again, consider the case of three 4-ohm voice coils and an output transformer which matches 4 ohms. This calls for a series-parallel arrangement with one of the elements in one series line a 4-ohm dummy load. (Refer to Fig. 47.)

In all of the foregoing cases, it was assumed that all of the speakers are to handle equal audio power. Many installations, however, demand an unequal power distribution; some speakers, as in a school classroom are required to handle only very low wattage, while speakers in an auditorium or gymnasium must be capable of greater power output. The assumption is made, of course, that a single program is to be heard in all these locations simultaneously. For example, consider the following: a 25-watt amplifier is to operate four speakers rated respectively at 15, 5, 3, and 2 watts, through a 500-ohm transmission line. The imped-



**Fig. 46.** The most typical multiple-speaker installations.



**Fig. 47.** A dummy load is used to obtain a proper impedance match.

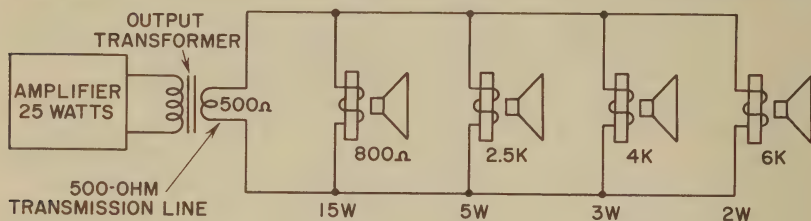
ances of the individual matching transformers may be determined from the following equation:

$$Z_P = Z_L \times \frac{P_A}{P_S} \quad (143B)$$

in which  $Z_P$  = the primary impedance of an individual transformer,  $Z_L$  = the line impedance,  $P_A$  = amplifier output power,  $P_S$  = corresponding speaker power. Substituting in Eq. 143B we have:

$$Z_P = 500 \times \frac{25}{15} = 833 \text{ ohms}$$

This is the required primary impedance of the transformer in-



**Fig. 48** Speakers of various impedances connected in parallel.

tended for use with the 15-watt speaker. The remaining speaker ratings (powers) may be substituted in sequence in order to obtain results of: 2500 ohms for the 5-watt speaker, 4170 ohms for the 3-watt speaker, and 6250 ohms for the 2-watt speaker. Odd impedances such as some of these are difficult to obtain except in custom-made transformers. A satisfactorily close match, however, is often realized by using standard transformers illustrated in Fig. 48.

## 19. Impedance Matching in Modulation Systems

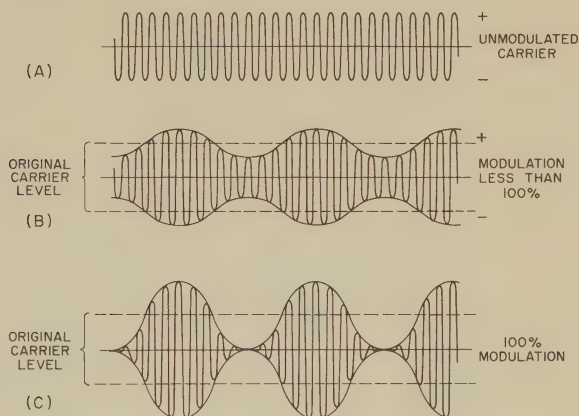
In all types of amplitude-modulation systems, an audio-frequency signal is heterodyned with an r-f carrier, causing side frequencies (sidebands) to exist above and below the r-f carrier. These side signals vary in amplitude and frequency in accordance with the modulating audio signal. There are many types of amplitude-modulating systems in use, all devoted to the production of sidebands.

The actual process of modulation, or heterodyning, is accomplished in an r-f amplifier stage. The gain of the r-f amplifier is made to vary at the audio-modulation frequency rate. Sidebands are then created equal to the instantaneous sum and difference of

the modulating and carrier frequencies. If all the sidebands, together with the carrier (the carrier at all times remaining constant in amplitude) are viewed simultaneously, a resulting r-f envelope is seen to vary in amplitude at the modulating frequency rate.

The receiver only makes use of the modulation, or *variations* in the envelope. An unmodulated carrier is shown in Fig. 49A.

**Fig. 49.** Modulated and unmodulated waveforms.



Since there are no amplitude variations, the receiver could recover no useful information from it. In Fig. 49B there is some amplitude variation, but maximum use is not being made of the carrier. The amplitude variations may be carried to a point where the envelope amplitude varies from zero to twice the carrier amplitude. This condition is called 100% modulation (Fig. 49C).

In Fig. 50, the gain and operating points of the r-f amplifier are made to vary at an audio-frequency rate by varying the amplifier's grid. The audio frequency amplifier (called the modulator) is a transformer coupled to the grid bias circuit of a class-C r-f amplifier. Audio-voltage variations in the secondary of the coupling transformer add algebraically to the fixed grid bias. During operation the bias is always changing, therefore the rectified grid current is also changing — effectively placing a variable load on the audio amplified tube. This effective impedance variation in the secondary makes it useless to state a given turns ratio to provide a proper impedance match between the modulator output tube and the grid of the r-f amplifier.

One of the most common means of impedance matching can be effected by placing a load resistance across the output transformer of such a low value as to completely mask the impedance variations contributed by the varying grid current. Under these conditions,

the modulator output tube “sees” an essentially fixed impedance. The power lost in this resistor, however, is not used in the circuit and the amplifier has to supply not only the requirements for 100% modulation but also for the load resistor (labeled R in Fig. 50). The value of this resistor should be approximately equal to the load into which the output tube should work. In this in-

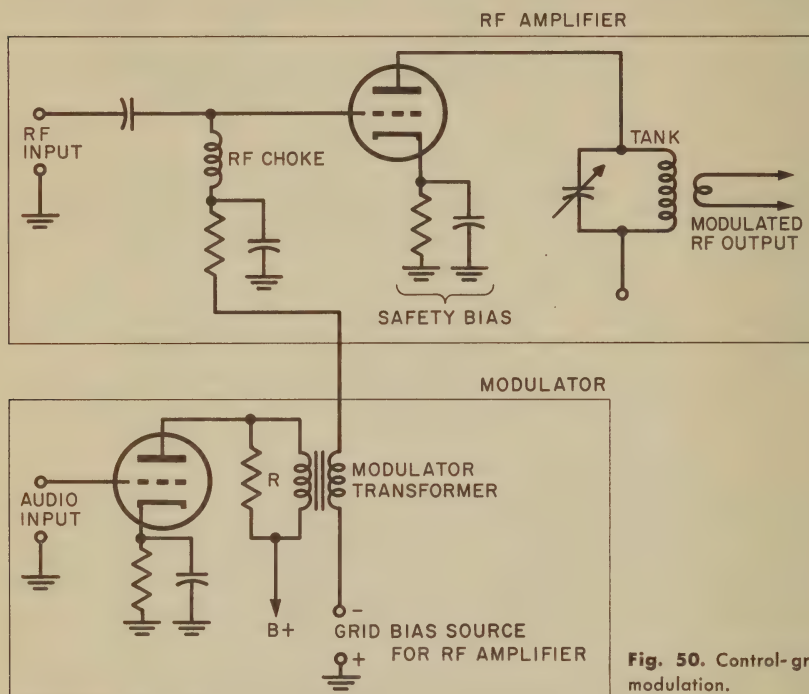


Fig. 50. Control-grid modulation.

stance of grid modulation, very little audio power is required because most of the excess power is dissipated in the resistor R. Hence the turns ratio of the matching transformer may be arbitrarily chosen as 1:1, with little ill effect.

In Fig. 51 the gain and operating point of the r-f amplifier are changed by varying the plate voltage of the r-f amplifier above and below a certain value. Again, this is accomplished by having the audio voltage that is present on the secondary of the modulating transformer combine with the static plate voltage of the r-f amplifier tube. The load impedance presented to the modulator is very nearly resistive. Hence we can say:

$$Z_m = R = \frac{E_b}{I_b} \quad (144)$$



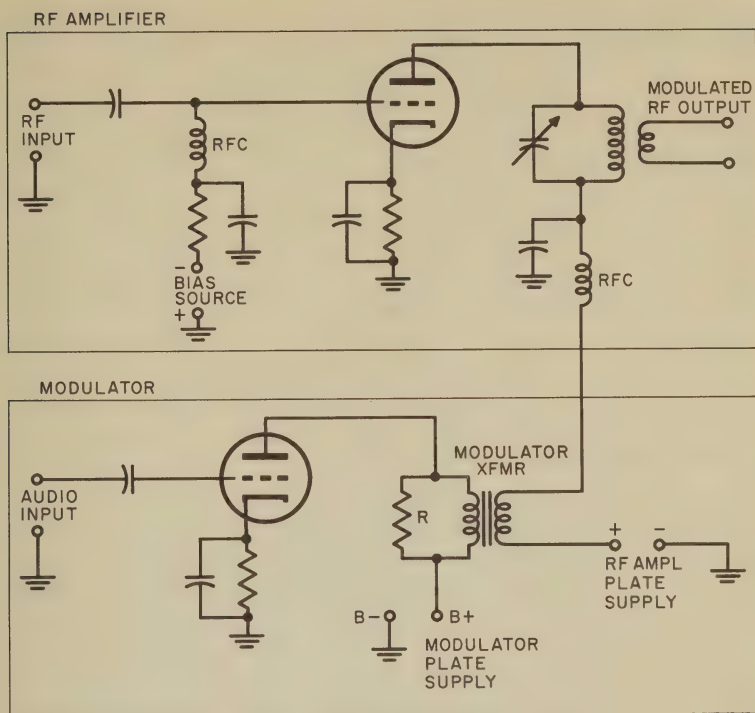


Fig. 51. Plate modulation.

where  $E_b$  is the plate voltage of the r-f amplifier in volts and  $I_b$  is the r-f amplifier plate current in amperes.

The required turns ratio of the impedance-matching transformer may be calculated from:

$$N = \sqrt{\frac{Z_p}{Z_m}} \quad (145)$$

Where  $N$  = the required primary-to-secondary turns ratio,  $Z_p$  = the proper plate-load impedance for the modulator output tube, and  $Z_m = \frac{E_b}{I_b}$  = the modulator load impedance ( $E_b/I_b$ ).

#### Illustrative

**Problem 23:** A pair of 2A3 low-power triodes are to be used as modulators for an r-f amplifier having a power input of approximately 30 watts. The plate voltage on the r-f amplifier is 900 volts. The 2A3's are to be operated in Class  $AB_1$  with fixed bias. Calculate the turns ratio of the modulation transformer.

**Solution:** According to the tube manual, push-pull 2A3's in Class  $AB_1$  (fixed bias) should work into a plate-to-plate load impedance of 3000 ohms. Thus,

$Z_p$  in Eq. 145 is 3000 ohms. At a power input of 30 watts and 900 volts on the plate, the load impedance is actually:

$$Z_m = \frac{E^2}{P} = \frac{(900)^2}{30} = 27,000 \text{ ohms}$$

Substituting these values into Eq. 145:

$$N = \sqrt{\frac{Z_p}{Z_m}}$$

$$N = \sqrt{\frac{3000}{27000}}$$

$$N = \frac{1}{3} \text{ or a turns ratio of 1:3}$$

## 20. Microphone-to-Line or Grid

A microphone is an energy converter that changes the mechanical movement of air particles (sound energy) into electrical energy. When the air particles are disturbed by a sound source, they strike the diaphragm of the microphone and by conversion cause the control of electron flow.

Microphones are of many designs. Each has some particular characteristics that make it best for use under certain circumstances.

Most microphones, with the exception of the carbon type, are inefficient. That is, the amount of sound energy going into the microphone is considerably greater than the electrical energy leaving it. This is important to remember because the first consideration in impedance matching is to conserve the small amount of power that is available at the microphone's output. Microphones are rated according to (1) frequency response, (2) sensitivity, and (3) impedance.

Frequency response gives dimension not only to the reproduction of the audio-frequency spectrum but also to the relative amplitude of the signal. To keep the response flat (amplitude unvarying), over a wide frequency range, some form of mechanical or electrical damping is often used. Damping in this instance is much akin to the load or swamping resistor used in modulating amplifiers. There are great differences in the impedances of various types of microphones. Crystal microphones (units using the piezoelectric feature of crystals) have impedances of several hundred thousand ohms. Microphones which use magnetic fields to convert energy range from a few ohms or even fractions of an ohm up to 500 ohms.

The sensitivity of a microphone is measured by its ability to convert sound into electrical energy. It is usually expressed in terms of the electrical power developed for a given sound input power.

Carbon microphones require direct current to actuate them. The output impedance is a few ohms, and varies according to the amount of dc passed. Because a line is usually higher in impedance than the microphone with which it is used, a step-up transformer must be used. The power available at the line terminals may be as great as  $-10$  dbm. The term dbm here refers to the db values in respect to one milliwatt of energy, therefore  $-10$  dbm means 10 db below one milliwatt, or 0.1 milliwatt. It usually is mentioned in respect to 600 ohms, but is not required if only power ratios are used. Carbon microphones, however, are not of good quality; their usual frequency range varies from 100 to 4000 cycles.

Dynamic microphones have a frequency response of from about 20 cycles to 9 kilocycles. The dynamic microphone is much lower in level (about  $-85$  dbm) than the carbon microphone. Its output impedance is also very low, less than 30 ohms.

Crystal microphones have extremely high impedance; several hundred thousands ohms. Some have response to frequencies as high as 17,000 cycles. The output is higher than for dynamic types of about 15 db.

Magnetic microphones have moving coils in a magnetic field, and their impedance is very low. In some of the ribbon-velocity types the impedance is as low as a few *thousandths* of one ohm. The output is also quite low, though the frequency response is excellent.

Clearly, then, there may be a wide variance in range of impedance, frequency response, and sensitivity from one microphone to another. Where the impedance is high, the microphone must be

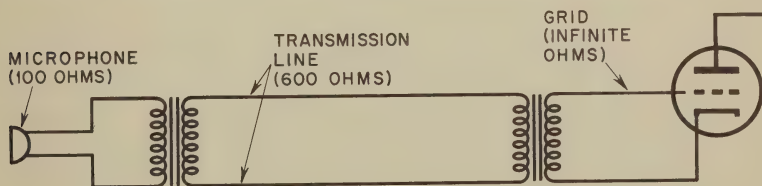


Fig. 52. A microphone-to-line-to-grid arrangement.

physically close to the circuit it is feeding or noise pickup will be great. Where the impedance of the microphone is higher than

the line impedance, it is desirable to use a transformer between the microphone and line to step down the impedance. This would be a simple impedance-matching problem except for the wide frequency range of many microphones. To accommodate these ranges, careful transformer design is necessary.

Figure 52 shows an arrangement of microphone-to-line-to-grid plus the impedances assumed for each step. The first transformer from microphone-to-line is between 100 and 600 ohms. The turns ratio can be computed from:

$$\frac{T_1}{T_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{100}{600}} = \text{approximately } \frac{1}{2.5} \quad (146)$$

Thus, the first transformer should have a turns ratio of 1:2.5 turns.

The second transformer is not quite so easy to compute. Let us construct an equivalent circuit to show the important aspects of the problem. This is given in Fig. 53.

The secondary of the transformer is open-circuited, so that the primary winding affects the line exactly as a simple inductance would. The circuit is therefore represented in the equivalent circuit by the inductive reactance  $X_p$  being the reactance in ohms of the primary winding. The leakage inductance is negligible and does not enter in this case.

The voltage at the grid is  $e$ , which is the voltage across the reactance  $X_p$  times the turns ratio. The value of  $e$  may then be described by the formula:

$$e = \frac{Z_p \times (\text{turns ratio}) \times E_L}{Z_L + Z_p} \quad (147)$$

Obviously the greater is  $Z_p$  the greater the voltage will be at the grid. This means that if the primary has a great deal of inductance the low frequency response will be satisfactory. The primary inductance is related to the geometry and material of the transformer, by:

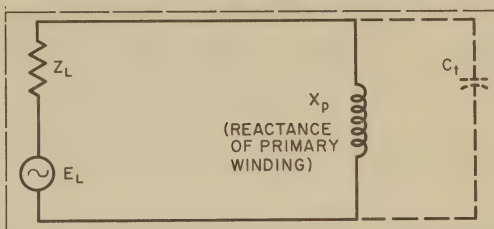
$$L_p = t_p^2 \left( \frac{d \Phi}{d t_p i_L} \right) \quad (148)$$

Here  $t_p$  is the primary turns,  $i_L$  the line current,  $\Phi$  the magnetic field. The portion in parentheses is the slope of the magnetization curve for the iron core of the transformer. The primary inductance can be made large either by making this slope large, which requires bulk, weight, and cost in the iron core, or by using a large number of turns in the primary.

The use of a large number of turns in the primary also requires a large number of turns in the secondary, and with the greater

turns of the secondary the distributed capacitance causes a loss at the higher frequencies. Capacitance which increases rapidly with the turns is represented in Fig. 53 by  $C_t$  in parallel with  $X_p$ . At a

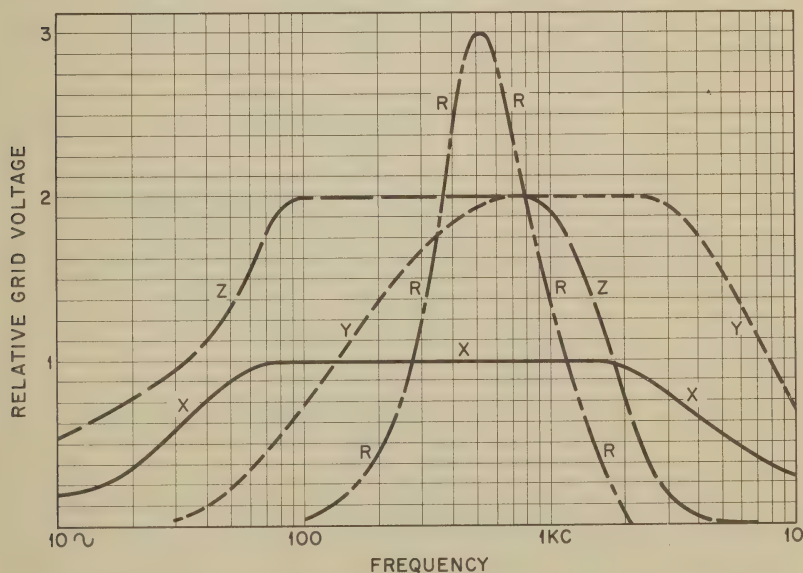
**Fig. 53.** A line-to-grid equivalent circuit.



sufficiently high frequency this capacitance effectively short circuits the input circuit causing the input voltage  $e$  to vanish.

We have seen that  $e$  vanishes at high frequencies because of  $C_t$ , and  $e$  vanishes at low frequencies because of  $X_p$ . This makes it necessary to select a value of inductance as well as turns ratio to make the bandwidth of the transformer great enough to satisfy the requirements of the circuit.

Figure 54 shows the effect of changing the number of turns of a transformer with an infinite grid load and a turns ratio, to produce one grid volt and no leakage reactance. Curve X is the normal



**Fig. 54.** A graph showing the effects of changing turns of a transformer.



curve or universal curve distribution for the problem. Curve Y shows the effect of increasing the turns ratio by decreasing the number of primary turns. This causes poor low-frequency response. Curve Z shows the effect of making a similar change in the turns ratio by using more secondary turns which causes poor high-frequency response. Curve R shows the effect of increasing the turns ratio by reducing the primary turns and increasing the secondary so that the secondary becomes a tuned resonant circuit with a sharp peak at whichever frequency  $X_p$  equals  $X_c$ .

## 21. Calculation of Loss Due to Mismatch

When a low-impedance microphone (up to about 300 ohms) is to be coupled to the grid of a vacuum tube amplifier, it is often important to know the order of the loss in db that may be expected from a given amount of mismatch between the microphone and the cable transformer.

The impedance relationships between the microphone and the transformer may be stated very simply: the best possible match should be obtained to assure the least loss in power transfer. When the microphone and cable are separated by a very short distance, the power loss due to mismatch may be calculated from the equation:

$$\text{db loss} = 10 \log_{10} \frac{(Z_s + Z_L)^2}{4Z_s Z_L} \quad (145A)$$

where  $Z_s$  is the source or microphone impedance and  $Z_L$  is the load impedance of the primary of the matching transformer.

The mechanism of this equation may first be tested by assuming a perfectly matched microphone and transformer system. In this case,  $Z_s = Z_L$ . Simplifying on this basis we have:

$$\begin{aligned} \text{db loss} &= 10 \log_{10} \frac{4Z^2}{4Z^2} \\ &= 10 \log_{10} 1 \text{ and since } \log 1 = 0 \\ \text{db loss} &= 0 \end{aligned}$$

Thus, when the impedances of the microphone and transformer are the same (and the connecting cable short), the loss due to reflection or mismatch is zero. To evaluate the effects of incorrect impedance matching, consider this example: A microphone having an internal impedance of 50 ohms works into the primary of

a transformer which acts as a load impedance of 100 ohms. What is the loss in db due to this mismatch?

In this problem,  $Z_s = 50$  ohms and  $Z_L = 100$  ohms. Substituting in Eq. 145A:

$$\begin{aligned}\text{loss in db} &= 10 \log_{10} \frac{(50 + 100)^2}{4(50 \times 100)} \\ &= 10 \log_{10} 1.13 \\ &= 0.53\end{aligned}$$

It should be noted that even with a mismatch of 2:1, as in this example, the loss in db would be hardly discernible. Thus, this indicates that considerable tolerance is permissible between the impedances of microphone and load insofar as power loss is concerned.

If the mismatch is more serious, say,  $Z_s = 30$  ohms and  $Z_L = 500$  ohms, the loss is considerably more serious. Thus:

$$\begin{aligned}\text{loss in db} &= 10 \log_{10} \frac{(30 + 500)^2}{4(30 \times 500)} \\ &= 10 \log_{10} 2.81 \\ &= 4.49\end{aligned}$$

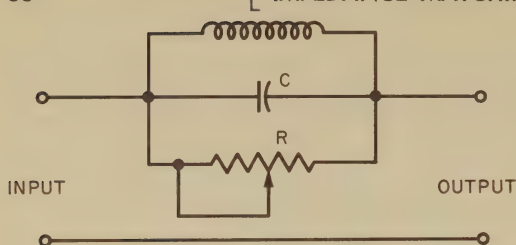
A loss of this magnitude, of course, would be readily detected and might be intolerable if reserve gain were not present.

## 22. Constant-Impedance Pads

Constant-impedance pads or attenuation equalizers (or just plain equalizers) permit correcting, at one point of a circuit, certain amplitude variations that otherwise would occur.

A long pair of wires which conducts a program signal from one location to another has sufficient shunt capacitance to appreciably reduce the signal at the end of the line. Unfortunately, this attenuation is not constant with frequency. Such a circuit which has an attenuation at 1000 cycles of 10 db might have an attenuation at 200 cycles of 6 db, and at 8000 cycles, 18 db. This variation of loss with frequency makes the program sound very "drummy" at the receiving end if not corrected. There is nothing that can be done to remove this shunt capacitance. However, it may be compensated for by the insertion of an equalizing circuit (Fig. 55).

This usually consists of a variable resistor connected across a parallel resonant circuit that has response characteristics which



**Fig. 55.** Simple equalizer circuit.

EQUALIZER HAVING GAIN CHARACTERISTICS  
WHICH COMPLEMENT LINE LOSSES

complement the loss characteristics of the line. In this case, the greatest losses occur at the higher frequencies due to the shunt capacitance of the line; the resonant point of the  $L/C$  circuit would then be adjusted so that, together with resistance  $R$ , the gain would gradually increase in the same proportion. By careful design, a simple filter of this variety can compensate for shunt capacitance effects.

If, for example, the circuit is intended for sending program material of up to 8000 cycles, the resonant circuit would be tuned to near 10,000 cycles. By adjusting the series resistance, the circuit can be made to respond equally well to all frequencies from some very low value, say 50 cycles, up to 8000 cycles. Thereafter the response will fall off, but up to the 8000 cycle point the response will be flat.

This corrective network is called an equalizer.

### Review Questions

1. Why is it important to match impedances at audio frequencies?
2. How large shall the load resistance be for a triode tube? For a pentode?
3. What factors determine the size of the load impedance?
4. What determines the amount of power that a transformer can handle?
5. Describe the role of an equivalent circuit.
6. Does the quality of reproduction have to be the same for background music in a club or restaurant as for a movie theatre?
7. What are the impedance problems involved in connecting several loud speakers to one amplifier?
8. What is modulating amplifier? Modulated amplifier?
9. Describe the use of a load resistor. A swamping resistor.
10. Is the impedance to be matched the same for all microphones?

## Chapter 4

# IMPEDANCE MATCHING AT RADIO FREQUENCIES

### 23. Areas of R-F Impedance Matching

The problem of matching impedances at radio frequencies is not a great deal different from the problems encountered at audio frequencies. The main differences are these: at radio frequencies, transmission lines and associated circuitry may approach a length comparable to the wavelength of the frequency being matched. Radiation losses (losses incurred through actual radiation of power from the matching or transmission device) become an important factor in the design of matching devices. In addition, insulation materials commonly used at audio frequencies no longer serve at radio frequencies. For these and other reasons, special techniques are often used for transferring power from one radio frequency device to another. For clarity, we will classify the problems of r-f impedance matching into two broad areas; inside the transmitter or receiver, and the antenna and transmission line circuitry. First we will review transmission theory.

### 24. Review of Transmission Lines

In Fig. 56A, an r-f generator or source is connected to a transmission line which is assumed to be of infinite length. The sinusoidal power emanating from the source will travel down the line toward infinity (advancing in phase  $2\pi$  radians for each wavelength covered) suffering a slight attenuation with each unit length of travel until finally no power is left to travel further. This infinitely long transmission line presents a certain load on the generator into which power enters but never returns. The impedance

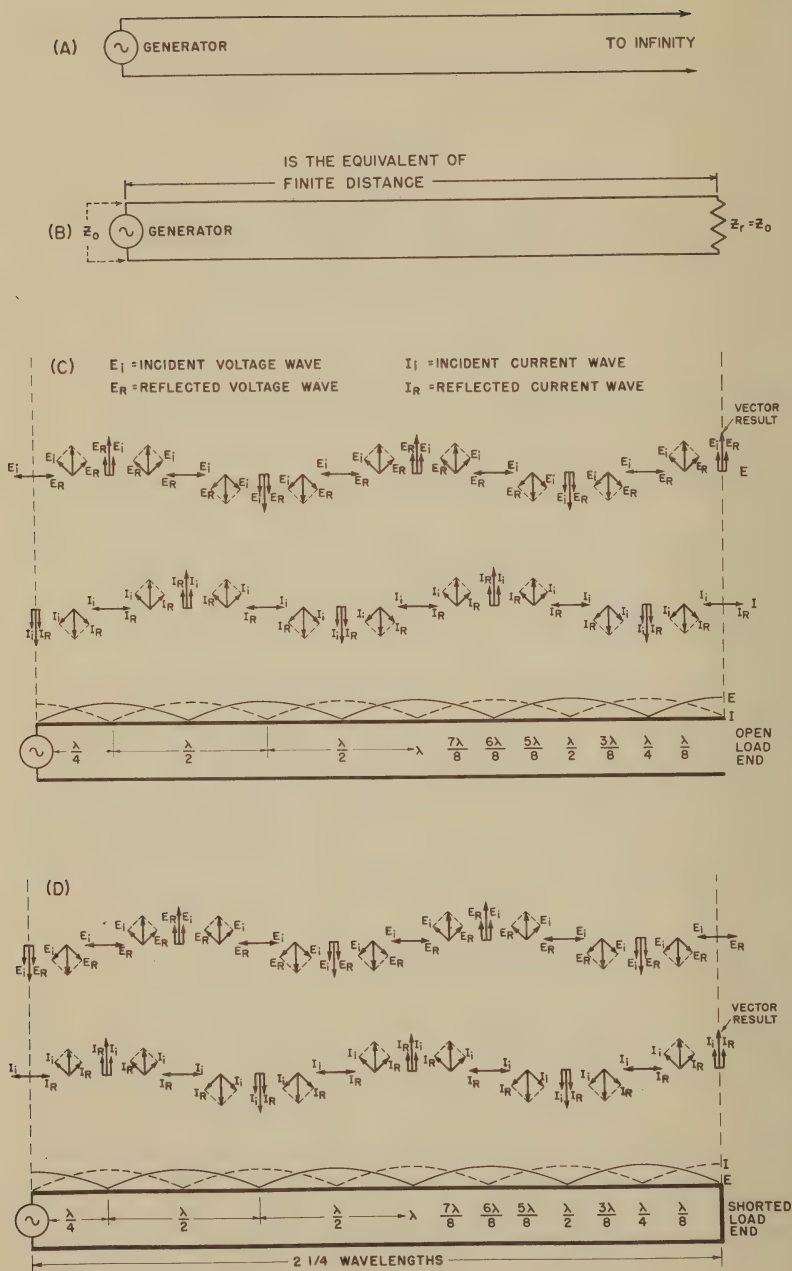


Fig. 56. A variety of transmission lines.



of this fictitious load is called the characteristic impedance of the transmission line ( $Z_0$ ). Instead of having an infinitely long line, suppose, as in Fig. 56B, there is a line of finite distance terminated at the end by a resistance equal in value to the load presented by the infinite line of Fig. 56A, namely, the  $Z_0$  of the line. Under these circumstances, as the energy travels down the line from the source and hits the load, it does not realize that this fixed resistance is not an infinite extension of the transmission line. Hence all of the energy arriving at this load resistance will be absorbed, and line and load will be impedance-matched.

If, however the load impedance, ( $Z_r$ ), does not equal the characteristic impedance, ( $Z_0$ ), not all of the r-f power arriving at the load will be absorbed. The part of this incident wave that is not absorbed is changed  $180^\circ$  in phase and sent back toward the source. This is called the reflected wave as opposed to the arriving incident wave. In Fig. 56C the voltage and current distribution on a line that is not matched is shown. This particular case, together with part D, illustrates an extreme condition of mismatched load and line. In Fig. 56C the load is an open circuit of a theoretical infinite impedance while in the latter case it is a short circuit with near zero impedance. Because the part of the incident wave not absorbed by the load suffers a  $180^\circ$ -phase reversal and is sent back towards the source, there is always a common phase relationship between the incident and reflected waves at any point on the line. In Fig. 56C it can be seen that at the load, after the incident wave suffers a phase reversal, there will be two vectors of opposite phase. In this instance no current can flow across the infinite load resistance, and no power will be absorbed; all of it is reflected. Hence the incident and reflected vectors are not only opposite in phase but equal in amplitude. As far as the reflected wave is concerned, the load is its source and it will be propagated back towards the source being retarded in phase by  $2\pi$  radians for every wavelength covered. The incident and reflected vectors are traveling in opposite directions and at the same velocity. Hence if we disregard the polarity of the actual sum vectors, load conditions will be repeated every half-wavelength and reversed every quarter-wavelength. The resultant sums (without polarity) of the current and voltage vectors are indicated as varying in amplitude at different points on the line. They are called standing waves, because they always have the same magnitude at any one point on the line. With an open-circuited load, voltage is maximum and current minimum at the load end. A quarter-wavelength back from the load towards the source, conditions are reversed and

voltage is minimum, current maximum. Points of maximum current or voltage are called current or voltage loops; points of minimum current or voltage are called nodes.

In the particular example cited, all the incident power was reflected and none absorbed. Now with other load conditions midway between an open and short, some of the incident power will be absorbed and some reflected. In this case the magnitude of the incident and reflected vectors will not be the same. Hence complete cancellation cannot exist at a current or voltage node, nor will a current or voltage loop have the magnitude that existed previously. The ratio between the magnitude of a voltage or current node and loop is called the standing wave ratio (SWR). Often the ratio of the voltage maximums to minimums is called the voltage standing ratio (VSWR).

The impedance at any point on the line is defined as the impedance that would be "seen" if the line were cut at that point and one were "looking" down the line towards the load. The actual impedance at this point is governed by the ratio of the voltage-to-current standing waves at the same point, that is

$$Z = \frac{E}{I} \quad (149)$$

Where  $Z$  = the impedance of the line at any point,  $I$  = the current of the line at any point, and  $E$  = the voltage of the line at any point.

Since the voltage and current standing waves vary along the line, the  $Z$  also varies. Of course in the special condition of a matched line the voltage and current do not vary (no reflections) and the impedance is constant. In this case, impedance equals the  $Z_0$  of the line. For a given length of line, the impedance presented to the generator is a function of the kind of load that is present at the end of the line. For instance in Fig. 56C the generator is located  $2\frac{1}{4}$  wavelengths from the load. At this point the current is maximum and the voltage minimum — equivalent to an impedance minimum. The generator will see under these conditions a short circuit. If the load were a short circuit such as illustrated in Fig. 56D, the generator would see an open circuit. If the load were of some resistive value between a short and open circuit and yet not equal to the  $Z_0$ , the generator would see some intermediate impedance value.

For a given load the impedance which the generator sees is a function of the line's length. For instance if the line in Fig. 56C were only 2 wavelengths long, the source would see an open circuit because this is the condition at the load and it is repeated

over multiples of a half-wavelength. In summary then, it can be seen that the impedance that the generator sees is a function of the distance of the load-to-source as well as the impedance value of the load. This control that the load has over the impedance that the generator sees is a fundamental principle of impedance matching with transmission lines and antenna circuitry. Let us examine how these principles are utilized.

## 25. The Quarter-Wavelength Line

The quarter-wavelength line is only a section of transmission line a quarter-wavelength long. Re-examining the impedance characteristics of this length of transmission line as shown in Fig. 57A and B, it can be seen that when there is zero voltage and impedance at the shorted end, the generator sees an open circuit. When the end is open-circuited, the generator sees a short circuit. This section of line then, is essentially an impedance-inverting device: a short circuit at one end is converted to an open circuit at the other end. This phenomena is not confined to a short and open circuit. If a low impedance exists at one end of the line, the other end looks like a high impedance to the generator. There is a direct mathematical relationship between the impedance at one end, the impedance at the other end, and the  $Z_o$  of the quarter wavelength line segment.

$$Z_o = \sqrt{Z_{in} \times Z_r} \quad (150)$$

Where  $Z_o$  = the characteristic impedance of the line segment,  $Z_{in}$  = the impedance looking "in" to the line segment, and  $Z_r$  = the load impedance of the line segment.

Using this simple equation, it is easy to solve an impedance-matching problem in Fig. 57C. Here the load impedance is only 52 ohms while the generator output impedance is 600 ohms. To match the two impedances so that a maximum amount of power will flow from the source to the load, it is necessary only to select a transmission line with the required characteristic impedance. In this instance the required impedance is:

$$Z_o = \sqrt{600 \times 52} = 177 \text{ ohms characteristic impedance.}$$

A close approximation of this impedance may be obtained by using a commercially available 150-ohm twin conductor transmission line. Such a line will convert 52 ohms to 430 ohms which is close to 600 ohms.

Perhaps a practical application of the quarter-wave line that is

more easily visualized is shown in Fig. 57D. Here a dipole with a center impedance at resonance of 72 ohms must be connected to a 600-ohm transmission line. Connecting the antenna directly to the 600-ohm line would result in something like an eight-to-one mismatch, with a consequent loss of power. Now if we connect

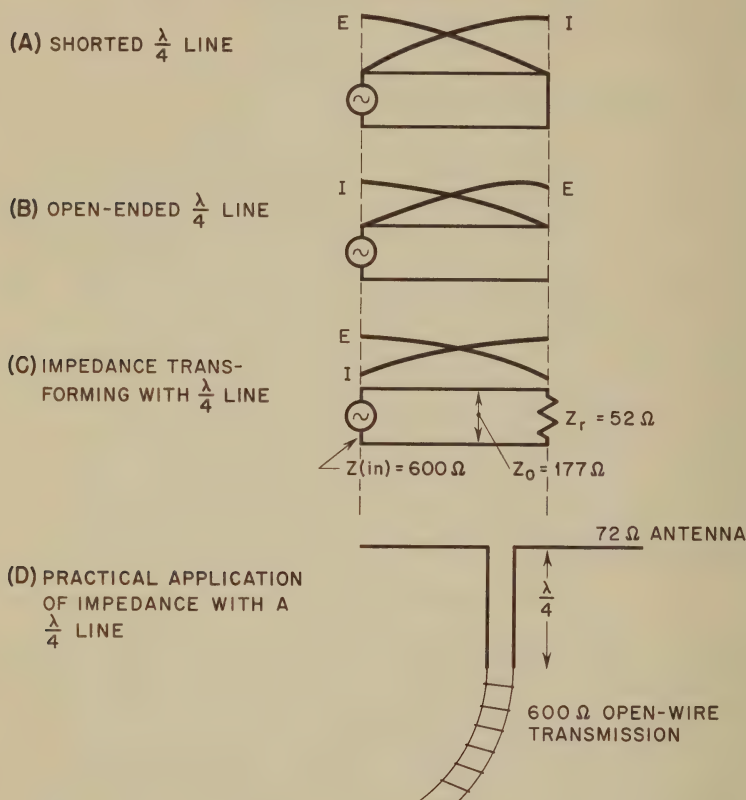


Fig. 57. Characteristics of quarter-wave sections.

a quarter-wavelength line segment of the proper  $Z_0$ , we can effect a practical match between the line and antenna. The impedance of this segment is:

$$Z_0 = \sqrt{600 \times 72} = 208 \text{ ohms} \quad (151)$$

Again, we could just as well use a commercial 150 ohm line. Such a line will convert 72 ohms to approximately 310 ohms.

There is a disadvantage in the above system of impedance matching in that a commercial transmission line of approximately the



correct impedance may not be available. A more practical method is illustrated in Fig. 57A. Here an open-ended quarter-wavelength line segment is connected to the input of a resonant dipole. A transmission line is connected to this line segment with movable contacts so that the point where the transmission line meets the segment may be adjustable. Although there is a mathematical method of defining the exact position of the stub, external factors often throw off this adjustment. In the end, one usually resorts to moving the contacts up and down the line segment until a match is affected. When the proper location is reached, the transmission line looks into an impedance equal to its own characteristic impedance. The antenna looks into a very low impedance. This impedance can be made to exactly equal the input impedance of the antenna by making the segment slightly longer than a quarter-wavelength.

It is significant to note in both of the above examples that it does not make any difference which end of the line segment is connected to the source and which end is connected to the load. In other words the impedance-transforming properties of this line segment have nothing to do with the direction of power flow. Therefore the antenna in Fig. 57D could have been a receiving or transmitting antenna. If the antenna were being used to receive power from space, it would act as the source of power for the line segment; if it were a transmitting antenna, the 600-ohm line would be the segment's source, while the antenna would be the load. This fact applies to nearly all impedance matching applications involving transmission line segments.

## 26. Balancing Units or "Balun" Transformers

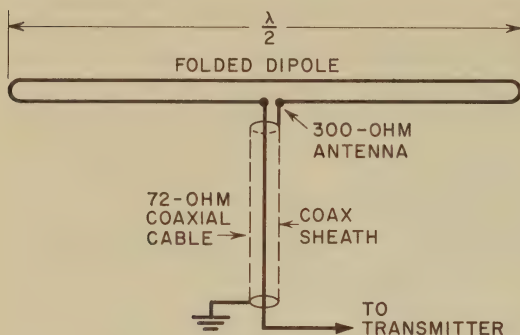
In many transmission line applications involving either receivers or transmitters, it is often desirable to transfer power from a balanced to an unbalanced system, or vice versa, without disturbing the balance. At the same time, a high percentage of power transfer demands an adequately close impedance match at the point of transfer.

There are several ways this may be accomplished, all of them involving matching sections of transmission lines or their equivalents.

Consider first the problem of matching a 72-ohm coaxial cable to a balanced, folded dipole antenna used as a transmitter radiator. Power is applied to the remote end of the coaxial cable by the transmitter-antenna coupling system which is grounded on



one side, making an unbalanced arrangement (Fig. 58). The center impedance of a half-wave folded dipole is 292.4, or approximately 300, ohms. Thus, the matching problem is two-fold: (a) to match an unbalanced line to a balanced antenna without de-



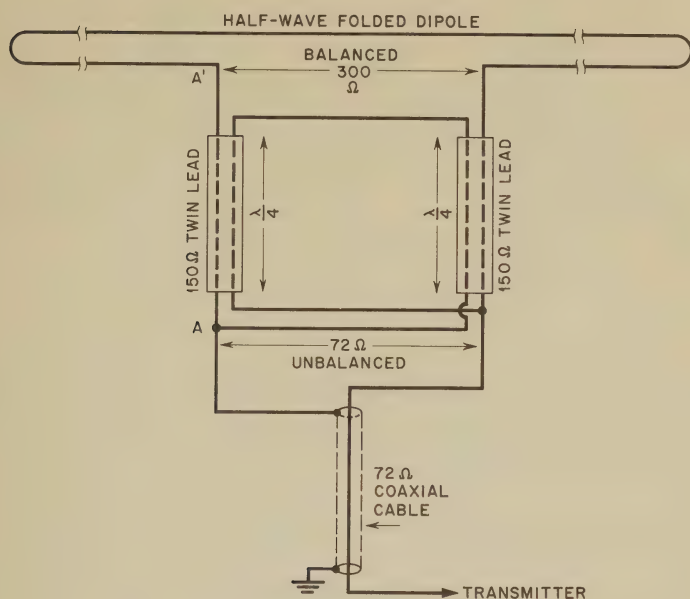
**Fig. 58.** Simple connection of cable to antenna.

stroying the symmetry of the antenna's balance to ground and (b) to transform impedances so that a reasonable match is obtained.

Both factors may be resolved by means of the balun arrangement illustrated in Fig. 59. Two quarter-wave sections of balanced transmission line such as twin-lead conductors are inserted between the 72-ohm coaxial cable and the balanced 300-ohm antenna input. At the input side, the lines are connected in parallel. If the quarter-wave sections are selected of 150-ohm characteristic impedance stock, then the paralleled lines present a load impedance of 75 ohms to the coaxial cable. This is a good impedance match but does not accomplish the second objective, that of preventing the unbalance in the input from affecting the operation of the balanced antenna.

On the output side, the quarter-wave section conductors are connected in series, resulting in an output impedance of 300 ohms which effectively matches the input impedance of the balanced, folded dipole. Since a quarter-wave line is interposed between the grounded section A of the coaxial feeder and the input to the antenna (point A'), there is complete isolation between the two. Hence, the grounding of A does not prevent A' from assuming any potential it wishes with respect to ground as demanded by the balanced system.

A balun works equally well in either direction. The system pictured in Fig. 59 might easily be altered to couple a balanced 75-ohm television receiving antenna through an unbalanced 72-ohm coaxial feeder to the standard balanced 300-ohm input of the television set. For such applications, the quarter-wave match-

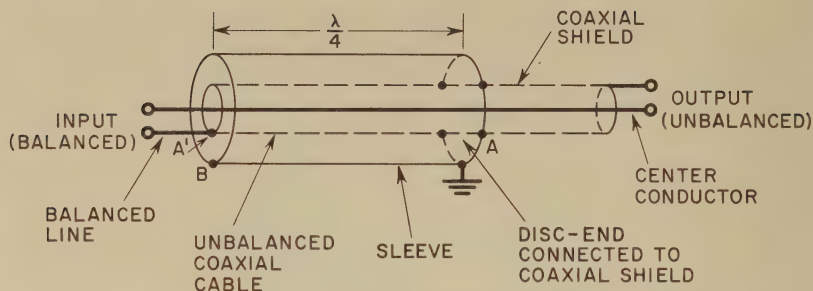


**Fig. 59.** Balun arrangement between cable and antenna.

ing sections are replaced by coils that are equivalent to the transmission line sections.

A second common balun arrangement utilizes a quarter-wave sleeve as shown in Fig. 60. In the arrangement in this drawing, the input voltage is applied across the two legs of the balanced line and the output is taken from the remote end of the unbalanced coaxial cable. A quarter-wave section in the form of a sleeve is connected to the coaxial shield as illustrated at one end and is left open at the other. Note that point A is grounded and it might be thought that the same must be true of Point A' since one is an extension of the other. The fact that the sleeve is a quarter-wave section, however, causes the impedance between points A' and B, the near portion of the sleeve, to approach infinity. Thus, the action of the sleeve completely isolates the unbalanced portion of the transmission line from the balanced section. By choosing the characteristic impedance of the coaxial cable of a value such that the proper impedance transformation occurs, both objectives are accomplished simultaneously.

At ultra-high frequencies, where the quarter-wavelength line segment is not too long to incorporate in the tube circuitry of a transmitter, these line segments are used to match the output



**Fig. 60.** Balun using quarter-wave sleeve.

tubes of a transmitter to a transmission line, as well as the transmission line to the input tube of a receiver. Some of these applications are illustrated in Figs. 61A through E.

## 27. Matching the Transmitter to the Line

The final amplifier tubes of a transmitter must work into a certain impedance. Almost without exception this impedance is much higher than the transmission-line impedance. At low frequencies, transmission-line segments are too long to be used as impedance transformers. Therefore an actual transformer is often used with the inductance of the primary of the transformer resonating with a variable capacitor and the output capacitance of the tube, to act as the final amplifier's plate-tank circuit. Several of these methods are shown in Fig. 61 together with its UHF counterpart. The theory of operation is: providing the load is a resistive impedance load, this impedance will be reflected right back to the plates of the output tube or tubes. In performing this, the resonant circuit acts as a simple transformer, for at resonance, reactive components are cancelled out in the tank circuit. If the tubes are connected across a high impedance point on the resonant circuit, while the line is connected to the proper low-impedance point, there will be an impedance step-down action, looking from the tubes towards the line. Selecting the proper impedances for a match is done by simply varying the position of the taps provided on the coil.

The above description was confined to examples where the load (transmission line) was a pure resistive impedance. This would be the condition when the line had no standing waves on it. It is, however, an illogical condition because only with very careful adjustment can all standing waves be eliminated. Much of the

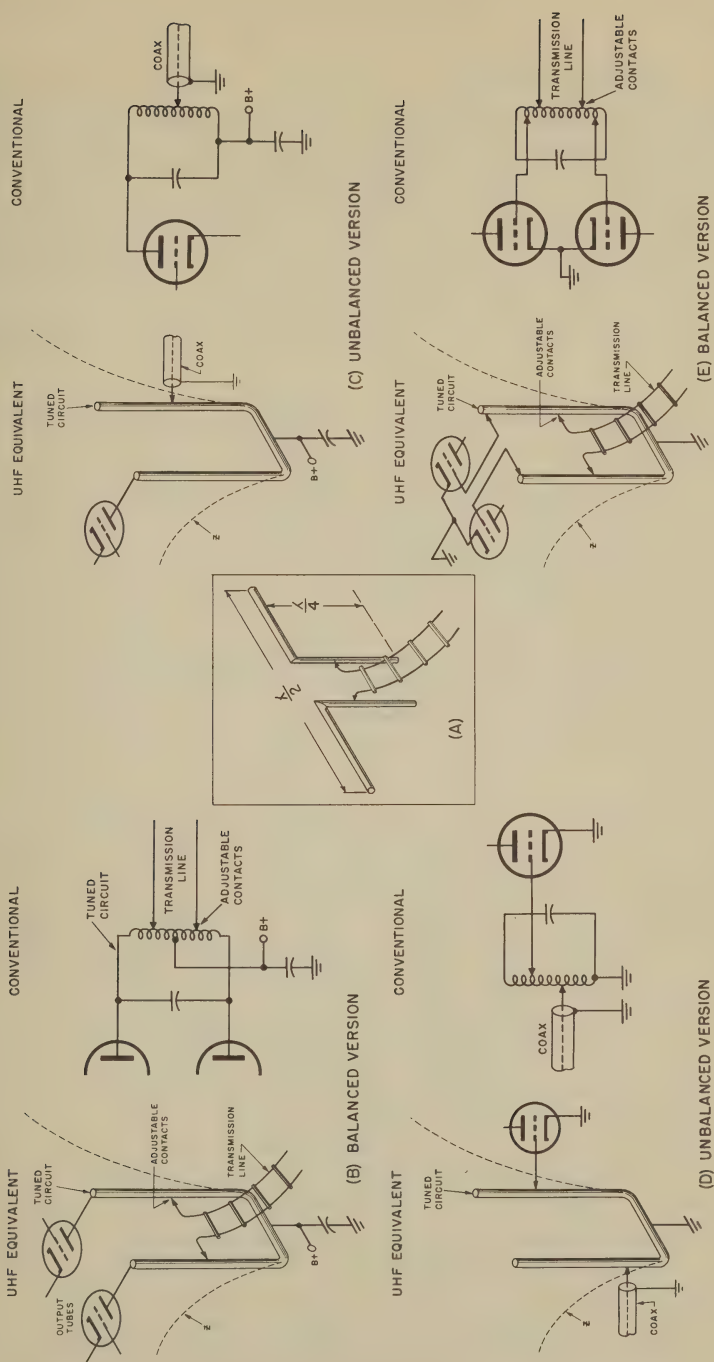
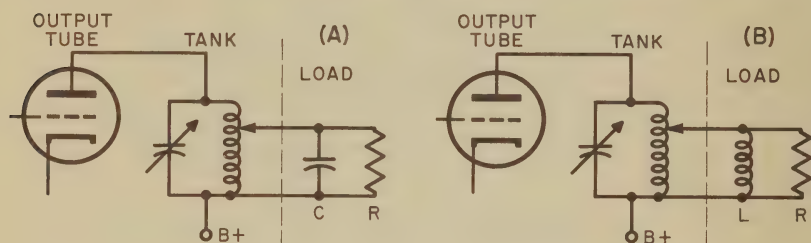


Fig. 61. Quarter-wave line sections used as matching transformer.

time transmission lines are run "tuned", that is, no attempt is made to match the impedance of the line-terminating load, as the line has excessive standing waves throughout its length. Any attempt, then, to match the output tube of a transmitter to the line must take into consideration the reactive as well as resistive component of the line's impedance, remembering that these will change with different lengths of transmission line. Consulting Fig. 62, it can be seen how this reactive-load impedance to the tank will reflect back into the tank and effect its resonant frequency.



LOAD INDUCTANCE OR CAPACITANCE APPEARS IN PARALLEL WITH TANK CIRCUIT

**Fig. 62.** Reactive loads affect the resonant frequency of the tube's tank circuit.

In part A the line exhibits a capacitive reactance and resistance. It may be a small resistance and a small reactance, but during the process of reflection, both of these are stepped up by the auto-transformer action of the tapped coil, thus throwing a large capacitive reactance and resistance across the output tube and in parallel with the tank resonant circuit. This will lower the resonant frequency of the tank circuit, and the tuning capacitor must be decreased in capacitance to bring the frequency back to where it was before the line was connected. The tap on the coil should be adjusted before and after the reactive compensation to insure a proper impedance match.

Figure 62B shows the condition of a line being inductive and resistive at the point of connection to the tank circuit. After adjustment of the tank tap, it will be noticed that the inductance of the load is thrown in parallel with the inductance of the tank. This lowers the effective inductance of the tank and the frequency rises. Increasing the value of the tuning capacitor will bring the tank back to frequency. It is axiomatic that the tuning procedure described above for lines with standing waves must be repeated

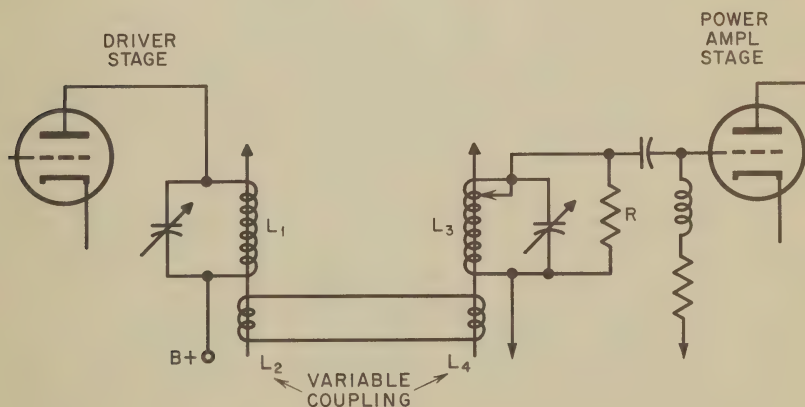


each time the length of the line is changed. In fact even if the line changes its physical shape (bends in the wind, etc.) the matching adjustments may have to be remade because the reactive and resistive components of the lines input  $Z$  presented to the tank circuit by the transmission line are a function of the *electrical length*, which can change even if the line comes only next to external objects.

## 28. Coupling Between Stages

Taking as our example again the final amplifier stage of a transmitter, it is necessary to couple energy into its grid from a previous driver stage. This driver stage may be an oscillator or an isolation stage called a buffer amplifier. Usually, in a transmitter, a simple transformer impedance-matching coupling device is not feasible since the two stages are physically separated by a number of inches or feet. Because of this problem the most popular method of coupling and securing the required impedance match is by a link-coupling system (Fig. 63).

If the distance between the two ends of the link is an appreciable fraction of a wavelength at the frequency used, the link must



IMPEDANCE MATCHING BY LINK COUPLING

**Fig. 63.** Schematic of impedances matched by link coupling.

be treated as a tuned or untuned (flat) transmission line. If the link is only a small fraction of the wavelength used, it may be considered only as a device to secure mutual coupling between

the driver and final tuned circuits. Only in the former must the effect of standing waves on the link be considered.

The transfer of power across a link coupling may be broken into two parts: the transfer of power from the driver plate tank into the link input coil, and the transfer of power from the link output coil to the grid tank of the final amplifier. The problems associated with coupling power from the driver to the link were discussed when we spoke of coupling power from the final amplifier to the transmission line. The only difference is, in this instance variable coupling between primary and secondary is obtained by varying the mutual inductance of  $L_1$  and  $L_2$ , as opposed to varying the turns ratio between the two coils.

If the system is to be operated on one frequency, and radiation from the link is of little consequence, then a resonant system may be used. Factors controlling this resonance are: the distributed capacitance of the line and coils, the number of turns on the input and output coils, and stray capacitances of the link system to ground. After the input and output coils have been chosen to produce the required resonant frequency, their proximity to the driver output and power amplifier input may be varied for the greatest transfer of power. A disadvantage of this resonant method is that voltage breakdown might occur at high-impedance points on the line. In addition, significant losses may occur through IR losses caused by high current existing at voltage nodes (low-impedance points).

If the frequency is to be varied, or if IR or radiation losses prohibit the use of a resonant system, the link must be made "flat" or free of standing waves. This nonresonant condition can occur only when the link-transmission line terminates in a resistance equal to its  $Z_0$ .

Referring to Fig. 63, the resistance marked R across the grid tank of the power amplifier is the equivalent input resistance of the tube. To make the line terminating in  $L_4$  "see" a resistance,  $L_4$  is made resonant at the operating frequency (reactance is canceled) and the grid resistance of the tube is reflected back into  $L_4$  so it resembles the proper terminating resistance for the transmission line. Now capacitive reactance is needed to cancel the inductive reactance of  $L_4$ , and thus produce resonance. This reactance is supplied by a slight detuning of the grid-tank circuit to make the tank inductive. After being reflected through the transformer  $L_3/L_4$ , it appears in  $L_4$  as a capacitive reactance of sufficient magnitude to resonate  $L_4$ . If the turns ratio between  $L_3$  and  $L_4$  is correct for a given coefficient of coupling, the grid resistance will appear in  $L_4$  with the proper magnitude.

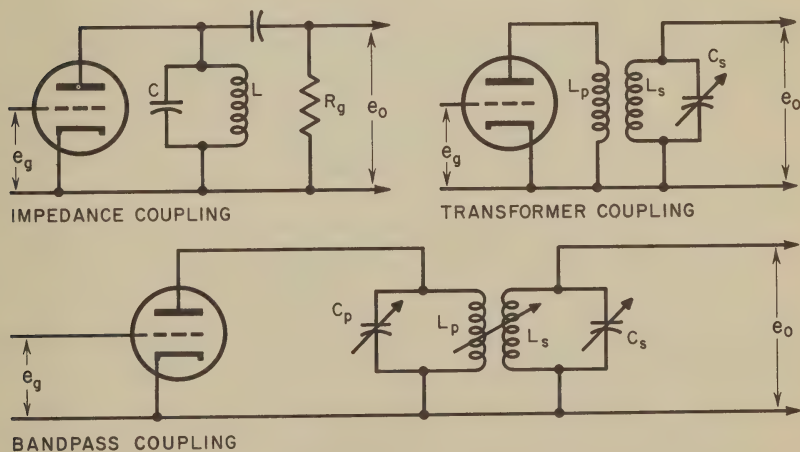
Adjustment of this type of coupling system requires a *Standing*

*Wave Ratio Indicator.* With this device connected to indicate when the line becomes "flat",  $L_4$  is set to a certain proximity with  $L_3$  and then the L-C ratio of the grid tank is varied. This changes the  $Q$  of the circuit, altering the magnitude of the tank resistance which is in parallel with  $R$ . Changing the L-C ratio, then, changes the total input resistance of the tube. Through this method we can control the magnitude of reflected resistance to the coil  $L_4$ . Maximum coupling between the driver and power amplifier for a given proximity of  $L_3$  and  $L_4$  will occur when the L-C ratio of the power amplifier grid tank circuit has a resistance parallel with  $R$ , and reflects a resistance into  $L_4$  equal to the  $Z_0$  of the link. This adjustment is strictly empirical, and to secure the greatest degree of coupling, the procedure should be repeated several times with various distances between  $L_4$  and  $L_3$ .

## 29. Other Network Arrangements for Radio Frequencies

There are many other arrangements that are used to couple stages of amplifiers that pass radio frequencies. Some common ones are shown in Fig. 64.

The circuits shown are used in receivers where these might be



**Fig. 64.** Representation of the coupling of radio-frequency stages.

called tuned radio-frequency or intermediate-frequency coupling stages.

The important characteristics of a tuned voltage amplifier are:

the amplification that can be obtained at resonance, the changes in amplification that can be obtained in the region near resonance, and the way the amplification alters when the resonant frequency of the tuned circuit is varied.

Tuned circuits using only a single resonant circuit to provide load impedance have a curve of amplification versus frequency that approximates the shape of an ordinary resonance curve, with a  $Q$  somewhat lower than the actual  $Q$  of the resonant circuit by itself. It is possible to describe the amplification and the effective  $Q$  of the amplification curve by the formula:

$$\text{Gain at resonance} = \frac{G_m \omega L Q}{1 + \frac{\omega L Q}{R_p} + \frac{\omega L Q}{R_g}} \quad (152)$$

where  $L$  is the inductance in the plate circuit,  $Q$  is the  $Q$  of the tuned circuit,  $\omega$  equals  $2\pi f$ ,  $R_p$  is the plate resistance, and  $R_g$  is the grid resistor in ohms.

$$\frac{\text{Effective } Q \text{ of amplification curve}}{\text{Actual } Q \text{ of tuned circuit}} = \frac{1}{1 + \frac{\omega L Q}{R_p} + \frac{\omega L Q}{R_g}} \quad (153)$$

The above formulas apply to the first of the circuits shown in Fig. 64. For the second circuit the formulas would be:

$$\text{Gain} = \frac{g_m \omega M Q}{1 + \frac{(\omega M)^2}{R_s R_p}} \quad (154)$$

$$\text{Ratio of } Q = \frac{1}{1 + \frac{(\omega M)^2}{R_s R_p}} \quad (155)$$

These formulas assume that  $R_p$  is much greater than  $\omega L_p$ . And for the third circuit:

$$\text{Gain} = g_m \omega M Q \quad (156)$$

$$\text{Ratio of } Q = \frac{1}{g_m \omega M Q} \quad (157)$$

In these circuits the purpose is not to get maximum power transfer but greatest input voltage to the next tube. The circuits are included only to give a notion of circuitry in radio-frequency coupling circuits.

**Review Questions**

1. List the main differences between impedance-matching problems at audio versus radio frequencies.
2. What is characteristic impedance? Does it depend upon the length of a transmission line?
3. What is a voltage or current loop? A node?
4. What is the name given to the ratio of magnitude between a voltage loop and node?
5. Why aren't transmission-line matching sections used at low radio frequencies?
6. What are link coupling circuits used for? Why are they necessary in some types of transmitters?



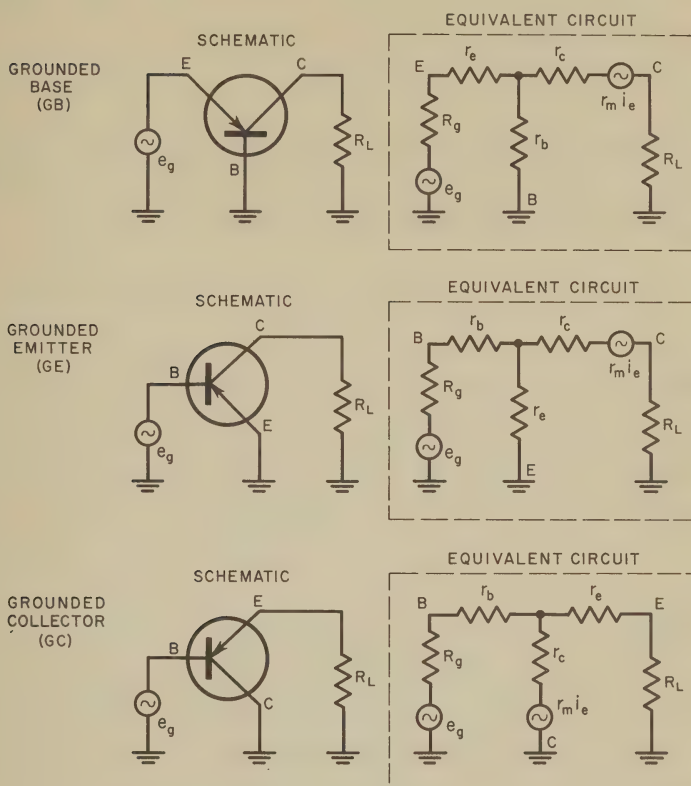
## Chapter 5

# IMPEDANCE MATCHING IN TRANSISTOR CIRCUITS

### 30. Transistor Impedance Considerations

Transistors as electronic devices are subject to the same physical laws that govern vacuum tubes in the sense that impedance-matching considerations apply to them as well as to tubes. Since the scope of this book does not permit a discussion of basic transistor theory, the reader is referred to standard textbooks on this subject now available. As an introduction to the subject of impedance matching in transistors, however, we will review briefly the standard transistor amplifier configurations before discussing impedance-matching problems and their solutions.

Transistors may be connected as amplifiers in three ways: grounded- or common-base connection (gb), grounded- or common-emitter connection (ge), and grounded- or common-collector connection (gc). These, together with their equivalent circuits, are shown in Fig. 65. With reference to transistor symbology, it should be noted that d-c biases have been omitted from the circuit diagrams in the interests of simplicity, and that impedances are referred to as resistances symbolized by  $r$ . It should be pointed out that a ratio of an observed voltage to an observed current in a transistor is really an impedance rather than a resistance. In circuit design work with transistors it is found that insignificant errors are introduced by assuming these to be resistance in most cases because the reactive component is so small compared to the resistive component. There are instances, however, where transistors *are* frequently sensitive due to the reactive component, small as it is. In such cases, the engineer or designer must make the corrections required to convert  $r$ 's to  $z$ 's in accordance with the applicable and approved methods. Since this factor does not



**Fig. 65.** Three ways transistors may be connected as amplifiers, and their equivalent circuits.

enter into a general discussion of impedance matching, it will be ignored in this chapter.

In Fig. 65 the meanings of the symbols may be listed as follows:

- $R_g$  resistance of the input device or signal generator
- $e_g$  signal input voltage
- $r_e$  emitter resistance
- $r_c$  collector resistance
- $r_b$  base resistance
- $R_L$  resistance of the output load
- $r_m i_e$  the generator or voltage source required in the equivalent circuit to indicate that this is an active rather than a passive network. An equivalent circuit for a transistor amplifier cannot be shown satisfactorily as a passive net-

work and must include a voltage source for adequate treatment

$r_m$  mutual resistance of the network

$i_e$  current through the emitter resistance  $r_e$ .

As indicated in the fundamental schematic diagrams of Fig. 65, primary consideration is given to junction transistors. The equations to be presented in the next section, however, are equally applicable to both junction and point-contact types.

### 31. Input Resistance of G-B, G-E, and G-C Connections

The input resistance of any transistor configuration is the resistance the external generator  $e_g$  sees as it looks toward the transistor and its associated components. In the g-b equivalent circuit in Fig. 65, everything to the right of the generator and its internal resistance  $R_g$  may be considered as  $R_i$ , the input resistance. Replacing the configuration by  $R_i$  as in Fig. 66, we can write:

$$R_i = \frac{e_g}{i_1} - R_g \quad (158)$$

The input loop current  $i_1$  can be found by applying Kirchhoff's Laws to the circuit parameters. Such a derivation would be out of place in this book but the final equation is given below to demonstrate that with it we may obtain the input resistance in terms of the circuit parameters.

$$i_1 = \frac{-e_g(r_b + r_m)}{(R_g + r_b + r_e)(R_L + r_b + r_e) - r_b(r_b + r_m)}$$

Substituting this value of  $i_1$  in Eq. 158, the input resistance for the g-b connection is found to be:

$$R_i = r_b + r_e - \frac{r_b(r_b + r_m)}{R_L + r_b + r_e} \quad (160)$$

This equation may be simplified by making the entirely justifiable assumption that  $r_b$  may be ignored since it is very small by comparison with  $r_m$ . In the case of a junction transistor, for example, a typical value for  $r_b$  might be 500 ohms as contrasted with almost 1 megohm for  $r_m$ . Taking  $r_b$  as insignificantly small:

$$R_i = r_b + r_e - \frac{r_b r_m}{R_L + r_e} \quad (161)$$

as the input resistance for the g-b configuration.

If average values of the circuit parameters for both NPN- and

PNP-type junction transistors are substituted in Eq. 161 the reader will find that the input resistance of the g-b configuration falls within a range of about 25 to 100 ohms. Obviously, a transistor, when connected as a ground-base amplifier, is a low-input imped-

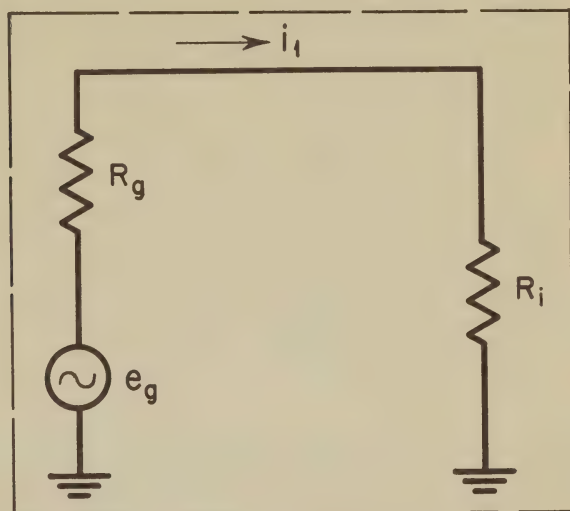


Fig. 66. Fundamental appearance of equivalent grounded-base circuit.

ance device and thus differs sharply from an ordinary vacuum tube amplifier where the input impedance may be well over 1,000,000 ohms at audio frequencies.

The input impedance of the g-e and g-c configurations may be computed with the assistance of similar equations and typical values obtained for each. All three are summarized in Table 6.

TABLE 6

Config- uration	Input Impedance Equation	Typical Values (Junction)
GB	$R_i = r_b + r_e - \frac{r_b r_m}{R_L + r_e}$	25 to 100 ohms
GE	$R_i = r_b + r_e + \frac{r_e(r_m - r_e)}{R_L + r_e + r_c - r_m}$	400 to 1000 ohms
GC	$R_i = r_b + r_c - \frac{r_c(r_c - r_m)}{R_L + r_e + r_c - r_m}$	700,000 ohms to 1 megohm

A set of representative data for an NPN transistor of current manufacture is given in Table 7. Typical characteristics at 25°C.

obtained at a test frequency of 1000 cycles, a collector voltage of 5 volts, and emitter current of 1 microampere are also presented. The reader may check these through with the equations.

TABLE 7

Collector resistance ( $r_c$ )	0.4 megohm minimum
Base resistance ( $r_b$ )	150 ohms
Emitter resistance ( $r_e$ )	22 ohms
Mutual resistance ( $r_m$ )	.96 megohms
Grounded-Base Connection	
Load impedance ( $R_L$ )	0.1 megohms
Input resistance ( $R_i$ )	35 ohms
Grounded-Emitter Connection	
Load impedance ( $R_L$ )	20,000 ohms
Input resistance ( $R_i$ )	480 ohms
Grounded-Collector Connection	
Load impedance ( $R_L$ )	500 ohms
Input resistance ( $R_i$ )	9500 ohms

32. Output Resistance of Transistor Amplifiers

The output resistance (or impedance) of a transistor is defined as the resistance presented to the load impedance  $R_L$  looking backward toward the source. The output resistance is an important characteristic because it determines in large measure what load impedance is required for good impedance matching. Output resistance may be obtained in terms of circuit parameters in very much the same manner as was used for input resistance. For the reader who wishes to investigate the relationship of output resistance to element and load resistances in typical transistors, Table 8 supplies the necessary equations:

TABLE 8

Connec- tion	Equation	Typical Values (Junction)
GB	$R_o = r_b + r_c - \frac{r_b r_m}{R_g + r_b + r_e}$	400,000 to 600,000 ohms
GE	$R_o = r_c + r_e - r_m + \frac{r_e(r_m - r_e)}{R_g + r_e + r_b}$	50,000 to 85,000 ohms
GC	$R_o = r_e + r_e - r_m - \frac{r_e(r_c - r_m)}{R_g + r_c + r_b}$	50 to 100 ohms



It is noteworthy that the figures given for input and output resistances of the g-c connection are somewhat analogous to the cathode-follower circuit of a vacuum tube. The high-input resistance and low-output resistance immediately suggest the same kind of step-down transformer action of which a cathode-follower is capable. From the point of view of impedance matching, it is evident that the g-c connection has important possibilities in circuits similar to those for which cathode-followers would be suitable.

### 33. Impedance-Match Considerations in Cascade Circuits

It is almost always possible, although it may often be difficult and inconvenient, to match the output impedance of a given amplifier stage (tube or transistor) to the input impedance of the stage that follows in a cascaded circuit with the aid of some impedance-transforming device. Yet the nature of some of the transistor configurations permits direct coupling without special attention to impedance transformers.

It is interesting to list the nine possible cascade arrangements together with their respective input and output impedances. Such a table illustrates at a glance those cascade possibilities in which impedances are naturally close to a good match.

**TABLE 9**

GB to GB	500,000 ohms to 100 ohms	mismatch
GB to GE	500,000 ohms to 1000 ohms	poor match
GB to GC	500,000 ohms to 700,000 ohms	good match
GE to GB	70,000 ohms to 100 ohms	mismatch
GE to GE	70,000 ohms to 1000 ohms	poor match
GE to GC	70,000 ohms to 700,000 ohms	fair match
GC to GB	75 ohms to 100 ohms	good match
GC to GE	75 ohms to 1000 ohms	poor match
GC to GC	75 ohms to 700,000 ohms	mismatch

Purely on the basis of impedance-matching considerations, five of the nine combinations (those labeled mismatch or poor match) do not seem to offer promising possibilities. On the other hand, it must be remembered that both input and output impedances of single-transistor amplifier stages are affected by the load resistance  $R_L$  and the generator resistance  $R_g$  respectively. This is evident at a glance from the equations presented in Tables 6 and 8. In cascaded configurations, the second stage behaves as the load resistance for the first and the first stage serves as the generator

impedance for the second, greatly altering the overall impedance picture.

When recalculations on this basis are performed, it is found that certain of these combinations in cascade do not represent serious mismatches and may be considered for use without circuit modifications or additional impedance transformers (junction transistors).

*Grounded base to grounded emitter* — fair voltage gain, good power gain, no serious mismatch. (Recommended)

*Grounded base to grounded collector* — poor voltage gain, fair power gain, serious mismatch. (Not recommended)

*Grounded emitter to grounded base* — excellent voltage gain, excellent power gain, good impedance match. (Highly recommended)

*Grounded emitter to grounded emitter* — excellent voltage gain, excellent power gain, good impedance match. (Highly recommended, even over the g-e to g-b connection)

*Grounded collector to grounded base* — poor voltage gain, poor power gain, very bad mismatch. (Not recommended)

*Grounded collector to grounded emitter* — good voltage gain, good power gain, good impedance match. (Especially recommended as a constant impedance-level device. Also has a much higher input impedance than the g-e to g-e cascade and offers superior impedance-matching features for high-impedance input devices such as crystal pickups, microphones, etc.)

*Grounded collector to grounded collector* — very poor voltage and power gain, very bad mismatch. (Not recommended)

In designing cascaded amplifiers, the selection of the input and output configurations is usually made before coupling methods are considered. In many instances, the choice is determined almost entirely by impedance considerations. As indicated by the block diagram in Fig. 67, the configuration used as the input stage must be chosen on the basis of the value of  $R_g$ , the generator impedance, and the connection used as the output stage on the basis of load impedance ( $R_L$ ). In the last analysis, the most important single factor is the stage gain. This is given by the equation (for a single stage):

$$G = \frac{4R_g R_L \alpha^2}{(R_g + r_i)^2}$$

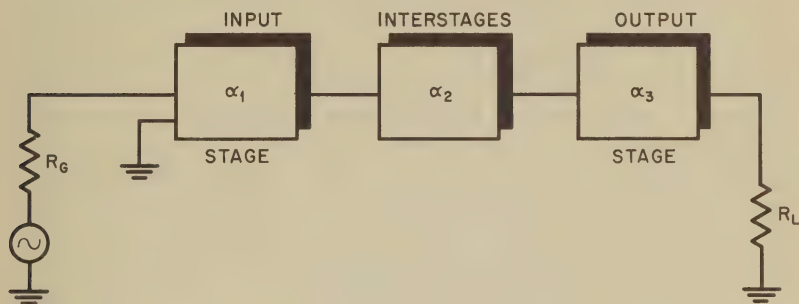
in which  $R_g$  is generator impedance,  $R_L$  is load impedance,  $\alpha$  is the current gain of the stage, and  $r_i$  is the input resistance of the configuration.

For cascaded stages, the gain of the first or input stage is a function of its current gain and generator resistance, and the gain of

the last stage dependent upon its current gain and the load resistance. For a three-stage amplifier (Fig. 67) the general gain equation becomes:

$$G = 4 \times \frac{R_g \alpha_1^2 \alpha_2^2 \alpha_3^2}{(R_g + r_1)^2} \times R_L$$

where  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  are the respective current gains for stage 1, stage 2, and stage 3. It may be seen from this equation that a cascade



**Fig. 67.** Calculation considerations in the design of transistorized cascaded amplifiers.

system has maximum overall gain when each stage is separately designed for greatest possible values of its associated gain factors.

On this basis, the configurations most suitable for input selection may be roughly determined as follows:

(a) If  $R_g$  is between 0 and 500 ohms, either a grounded-base or grounded-emitter connection is recommended.

(b) If  $R_g$  is between 500 and 1500 ohms, a grounded-emitter connection is recommended.

(c) If  $R_g$  is over 1500 ohms, either a grounded-emitter or grounded-collector is recommended.

Similarly, the choice of output configuration must be dictated by the value of the load impedance. Thus:

(a) If  $R_L$  is between 0 and 10,000 ohms, either a grounded-collector or grounded-emitter connection is recommended.

(b) If  $R_L$  is between 10,000 and 500,000 ohms, a grounded-emitter connection is recommended.

(c) If  $R_L$  is over 500,000 ohms, either a grounded-emitter or grounded-base connection is recommended.

Note that the grounded-emitter connection is mentioned as a possibility for each case. This might indicate that the choice of this configuration would be satisfactory under any circumstances. This is generally true, except that certain designs which involve

coupling networks or bias voltages that demand one of the other connections are often encountered. In such cases, consideration must be given to grounded-base and grounded-collector configurations.

### 34. Coupling Methods in Cascaded Amplifiers

*Low-frequency amplifiers* — Either r-c or transformer coupling is suitable for low-frequency amplifiers in and around the audio portion of the frequency spectrum. R-c coupling is favored between voltage amplifiers or between radio detectors and the first audio driver stage, because it is less expensive and lends itself to compact equipment design. Transformers are almost always used between audio drivers and output stages for two reasons: (1) the input and output circuits of the last stage must handle definite amounts of power and (2) the output stage of most transistorized equipment is almost always a push-pull circuit. Transformers by their very nature are better adapted to power and phase inversion applications than are r-c components.

Examples of these applications are illustrated in Fig. 68, which shows the detector and audio section of an all-transistor portable radio receiver. A 1N60 diode is used as a second detector following the intermediate-frequency amplifier section. The r-c coupling network to the input of the first transistor (T1310, audio driver) comprises the volume control, the coupling capacitor  $C_c$ , and the base resistor  $R_c$ . Interstage transformer  $T_1$  matches the collector resistance of the audio driver to the bases of the two 2N185 transistors.  $T_1$  also produces the phase inversion required for the operation of the push-pull output stage.  $T_2$  matches the collector impedance of the push-pull output stage to the voice coil of the speaker.

In connection with the r-c coupling network consisting of the volume control,  $C_c$ , and  $R_c$ , it is important to observe the values of the last two components. The capacitance of  $C_c$  is 12  $\mu\text{f}$ , a very large value indeed when compared to audio coupling capacitors used in vacuum tube equipment. The input resistance of the audio driver transistor in grounded-emitter configuration is in the order of 500 ohms. To avoid an excessively large signal-voltage drop across  $C_c$ , because of the low impedance of the transistor input, this capacitor must have a very small reactance to the lowest frequency to be amplified. This explains the large capacitance. It can also be shown that the interstage loss in gain is less than 1 db if  $R_c$  is made at least 10 times as large as the input



resistance of the transistor;  $R_c$  was selected to be well over 10 times greater than the  $r_i$  for this transistor.

Transformer  $T_1$  is designed to match the output impedance of the grounded-emitter audio driver (approximately 50,000 ohms) to the input impedance of the grounded-emitter push-pull output stage (approximately 1200 ohms). Type MT-8 would be suitable

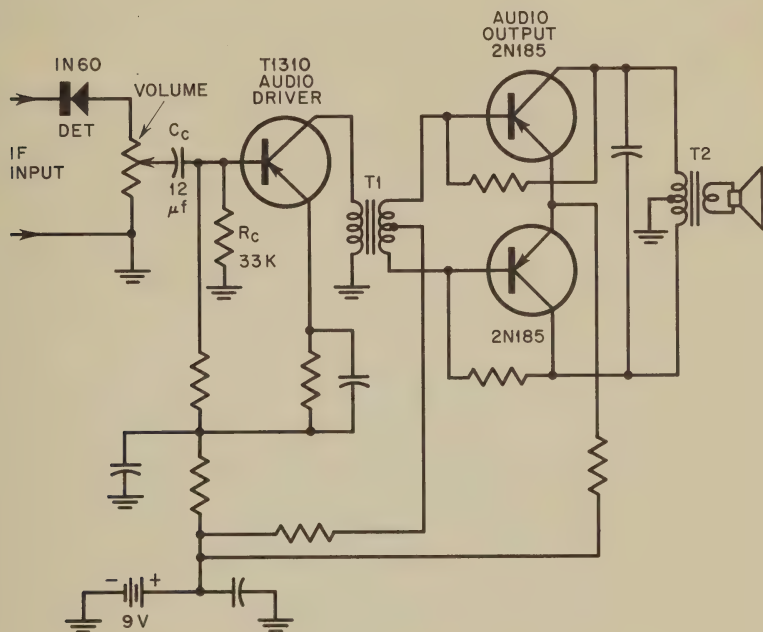


Fig. 68. Detection and audio section of CBS portable model 296.

for this application. Similarly, transformer  $T_2$  must match the output impedance of the push-pull stage (approximately 2000 ohms) to the speaker voice coil (16 ohms). Type TY-47X is designed to match impedances for a combination of this sort.

### 35. Intermediate Frequency Amplifiers

The standard vacuum tube i-f amplifier tube does not affect the design of the i-f transformers with which it works. That is, its input and output impedances are such that its loading effect on the i-f transformers need not be considered in design calculations. Grid impedances are extremely high and plate circuit im-



pedances, particularly in normal i-f pentodes, are usually well above 0.5 megohm.

As we have seen, this is not true of transistors. In the grounded-emitter connection, the collector impedance of a modern high-frequency transistor may be relatively high, in the order of 100,000 ohms at 455 kc, but the input impedance is quite low — between 300 and 500 ohms. Impedance matching is essential to secure maximum power transfer; furthermore, the bandpass characteristics of the i-f transformer cannot be determined without including the effects of transistor loading in the calculations.

Both single- and double-tuned i-f transformers are currently in use. In some cases, especially where adjacent channel rejection is not particularly important, single tuned transformers are preferred because they have less insertion loss than double-tuned types. Considering the relatively limited gains available from high-frequency transistors, insertion losses may be of great importance. Double-tuned transformers, however, are almost always found in equipment in which good adjacent-channel rejection characteristics are required.

As an example, an overwhelming number of recent portable transistor radios employ single tuned transformers as illustrated in Fig. 68. In this way, the insertion loss is maintained at a relatively low figure; the adjacent-channel rejection requirements are more than satisfied by single-tuned transformers in this application.

It should be noted that impedance matching is accomplished by taking into consideration: (a) the  $X_L$  of the transformer primary winding required to resonate with a given value of tuning capacitance and (b) the image impedances seen by the output of the driving transistor and the input of the driven transistor. The primary winding of the transformer is tapped at that position which insures the best possible match between both transistors and the coupling transformer.

Assuming that the collector output impedance is 100,000 ohms, the tap on the primary is adjusted so that the number of turns on the coil between the tap and signal-ground presents an approximate impedance of 100,000 ohms to the transistor. The number of turns in the untuned secondary winding is calculated to permit matching the input impedance of the next transistor, perhaps 500 ohms.

### Review Questions

1. Name three methods of connecting transistors.
2. Define the output resistance (or impedance) of a transistor.
3. Which type of transistor connection is similar to the cathode follower circuit?

4. Which cascade arrangements give excellent results?
5. What type coupling is preferred for (a) Audio voltage amplifiers? (b) Audio power amplifiers?
6. Why do we need impedance matching when using transistors at RF frequencies?
7. Which type of IF transformer is preferred? Why?
8. Which factors must be taken into consideration when impedance matching IF transformers?

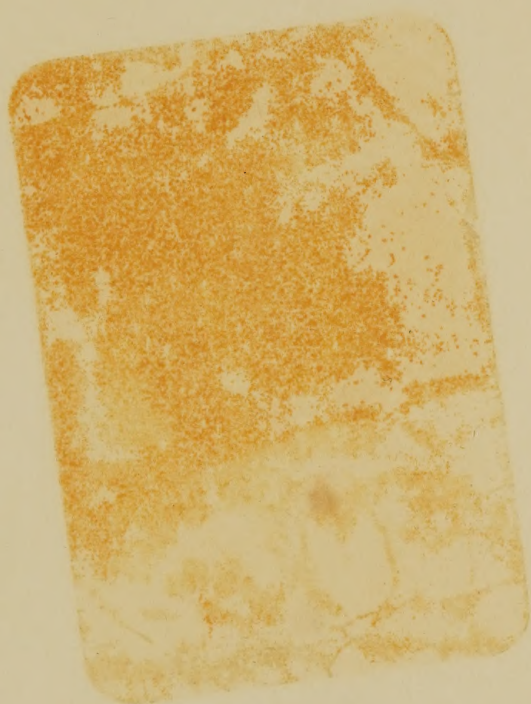
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